HYPOTHESIS TESTING GUIDE

Common Symbols
- \( n \) = Sample Size
- \( \bar{x} \) = Sample Mean
- \( s \) = Sample Standard Deviation
- \( s^2 \) = Sample Variance
- \( \hat{p} \) = Sample Proportion
- \( r \) = Sample Correlation Coefficient
- \( \alpha \) = \( P(\text{Type I error}) \)

- \( N \) = Population Size
- \( \mu \) = Population Mean
- \( \sigma \) = Population Standard Deviation
- \( \sigma^2 \) = Population Variance
- \( p \) = Population Proportion
- \( \rho \) = Population Correlation Coefficient

Yes

Is \( \sigma \) known?

No

Use the \( z_{\alpha/2} \) values
and \( \sigma \) in the formula.*

Use the \( t_{\alpha/2} \) values
and \( s \) in the formula.*

*If \( n < 30 \), the variable must be normally

Look for these key words to help set up your hypotheses:

<table>
<thead>
<tr>
<th>Two-tailed Test</th>
<th>Right-tailed Test</th>
<th>Left-tailed Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 : \mu = \mu_0 )</td>
<td>( H_0 : \mu = \mu_0 )</td>
<td>( H_0 : \mu = \mu_0 )</td>
</tr>
<tr>
<td>( H_1 : \mu \neq \mu_0 )</td>
<td>( H_1 : \mu &gt; \mu_0 )</td>
<td>( H_1 : \mu &lt; \mu_0 )</td>
</tr>
</tbody>
</table>

Claim is in the Null Hypothesis

<table>
<thead>
<tr>
<th>=</th>
<th>( \leq )</th>
<th>( \geq )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is equal to</td>
<td>Is less than or equal to</td>
<td>Is greater than or equal to</td>
</tr>
<tr>
<td>Is exactly the same as</td>
<td>Is at most</td>
<td>Is at least</td>
</tr>
<tr>
<td>Has not changed from</td>
<td>Is not more than</td>
<td>Is not less than</td>
</tr>
<tr>
<td>Is the same as</td>
<td>Within</td>
<td></td>
</tr>
</tbody>
</table>

Claim is in the Alternative Hypothesis

<table>
<thead>
<tr>
<th>( \neq )</th>
<th>( &gt; )</th>
<th>( &lt; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is not</td>
<td>More than</td>
<td>Less than</td>
</tr>
<tr>
<td>Is not equal to</td>
<td>Greater than</td>
<td>Below</td>
</tr>
<tr>
<td>Is different from</td>
<td>Above</td>
<td>Lower than</td>
</tr>
<tr>
<td>Has changed from</td>
<td>Higher than</td>
<td>Shorter than</td>
</tr>
<tr>
<td>Is not the same as</td>
<td>Longer than</td>
<td>Smaller than</td>
</tr>
<tr>
<td>Bigger than</td>
<td>Decreased</td>
<td></td>
</tr>
<tr>
<td>Increased</td>
<td>Reduced</td>
<td></td>
</tr>
</tbody>
</table>

The rejection rule:
- p-value method: reject \( H_0 \) when the p-value \( \leq \alpha \).
- Critical value method: reject \( H_0 \) when the test statistic is in the critical tail(s).
- Confidence Interval method, reject \( H_0 \) when the hypothesized value found in \( H_0 \) is outside the bounds of the confidence interval.
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One Sample Tests:

Means

\( H_0: \mu = \mu_0 \)  \( H_1: \mu \neq \mu_0 \)

Test statistic when \( \sigma \) is given in the problem: 
\[
    z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}
\]

Test statistic when \( \sigma \) is unknown: 
\[
    t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
\]

- Calculator Short Cut: Press the [STAT] key, go to the [TESTS] menu, arrow down to either the [Z-Test] or [t-Test] option and press the [ENTER] key. Arrow over to the [Stats] menu and press the [ENTER] key. Then type in value for the hypothesized mean \( (\mu_0) \), standard deviation, sample mean, sample size, arrow over to the \( \neq, <, > \) sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and p-value.

- Or (If you have raw data in a list) Select the [Data] menu and press the [ENTER] key. Then type in the value for the hypothesized mean \( (\mu_0) \), type in your list name (TI-84 L_1 is above the 1 key) (TI-89 list names are in the main folder under Var-Link), leave Freq 1 alone, arrow over to the \( \neq, <, > \) sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and the p-value.

Proportions

\( H_0: p = p_0 \)  \( H_1: p \neq p_0 \)

Test statistic is 
\[
    z = \frac{\hat{p} - p_0}{\sqrt{p \cdot q / n}} \text{ where } \hat{p} = \frac{x}{n}
\]

- Calculator Short Cut: Press the [STAT] key, select the [TESTS] menu, arrow down to the option [1-PropZTest] and press the [ENTER] key. Type in the value for the hypothesized proportion \( (p_0) \), X, sample size, arrow over to the \( \neq, <, > \) sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and p-value. Note: sometimes you are not given the X value but a percentage instead. The calculator will give you an error message if you put in a decimal for X or n. For example if \( \hat{p} = .22 \) and \( n = 124 \) then \(.22 \times 124 = 27.28 \), so use \( X = 27 \).
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Variance or Standard Deviation

\[ H_0: \sigma^2 = \sigma_0^2 \quad \text{or} \quad H_0: \sigma = \sigma_0 \]
\[ H_1: \sigma^2 \neq \sigma_0^2 \quad \text{or} \quad H_1: \sigma \neq \sigma_0 \]

Both use test statistic \( \chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \) with \( df = n - 1 \).

Two Sample Tests:
2 Means – Independent Populations

\[ H_0: \mu_1 = \mu_2 \quad \text{or} \quad H_0: \mu_1 - \mu_2 = (\mu_1 - \mu_2)_0 \]
\[ H_1: \mu_1 \neq \mu_2 \quad \text{or} \quad H_1: \mu_1 - \mu_2 \neq (\mu_1 - \mu_2)_0 \]

where usually you have \( H_0: \mu_1 - \mu_2 = 0 \) \( H_1: \mu_1 - \mu_2 \neq 0 \)

Test Statistic when \( \sigma_1^2 \) and \( \sigma_2^2 \) are given: \( z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \).

• Press the [STAT] key, select the [TESTS] menu, arrow down to the option [2-SampZTest] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu when stats are already given and press the [ENTER] key. Then type in the population standard deviations, the first sample mean and sample size, then the second sample mean and sample size, or the list names for raw data. Arrow over to the \( \neq, <, > \) sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic \( z \) and the p-value.

Confidence interval when \( \sigma_1^2 \) and \( \sigma_2^2 \) are given: \( (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \).

• Press the [STAT] key, arrow over to the [TESTS] (interval on the TI-89) menu, arrow down to the option [2-SampZInt] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu and press the [ENTER] key. Then type in the population standard deviations, the first sample mean and sample size, then the second sample mean and sample size, then enter the confidence level. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval.

Test statistic when \( \sigma_1^2 \) and \( \sigma_2^2 \) are unknown: \( t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \).

• Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [2-SampTTest] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu and press the [ENTER] key. Enter the list names or the means, sample standard deviations, sample sizes, and confidence level. Then arrow over to the not equal,  \( <, > \) sign that is in the alternative hypothesis statement then press the [ENTER] key. Highlight the No option under Pooled for unequal variances. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and the p-value.

with \( df = \left( \frac{1}{n_1 - 1} \left( \frac{s_1^2}{n_1} \right)^2 \right) + \left( \frac{1}{n_2 - 1} \left( \frac{s_2^2}{n_2} \right)^2 \right) \)

Confidence interval when \( \sigma_1^2 \) and \( \sigma_2^2 \) are unknown: \( (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \).

• Press the [STAT] key, arrow over to the [TESTS] (interval on the TI-89) menu, arrow down to the option [2-SampTInt] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu and press the [ENTER] key. Enter the list names or means, sample standard deviations, sample sizes, and confidence level. Highlight the No option under Pooled for unequal variances. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval.
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2 Means – Dependent Populations
Find the difference (d) between each matched pairs.

\[ H_0 : \mu_D = 0 \]
\[ H_1 : \mu_D \neq 0 \]

Test statistic: \[ t = \frac{\overline{d} - \mu_0}{s_d / \sqrt{n}} \]

- Type the data for group 1 in L1 and group 2 in L2. Arrow up so that you are highlighted on the label of L3. Then subtract the two lists L1 - L2. This puts the differences into L3. In the [STAT] menu, arrow over to the [TESTS] menu, arrow down to the option [T-Test] and press the [ENTER] key. Arrow over to the [Data] menu and press the [ENTER] key. Then type in the hypothesized mean as 0, List: L3, leave Freq:1 alone, arrow over to the ≠, <, > sign that is the same in the problems alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the t-test statistic, the p-value, \( \overline{d} = \overline{d} \) and \( S_x = S_D \).

Confidence Interval: \( \overline{d} \pm t_{\alpha/2} \left( \frac{s_d}{\sqrt{n}} \right) \) with df= n - 1.

- Type the data for group 1 in L1 and group 2 in L2. Arrow up so that you are highlighted on the label of L3. Then subtract the two lists L1 - L2. This puts the differences into L3. In the [STAT] menu, arrow over to the [TESTS] (interval on the TI-89) menu, arrow down to the [T-Interval] option and press the [ENTER] key. Arrow over to the [Data] menu and press the [ENTER] key. Type in the List: L3, Freq: 1, and the confidence level. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval, \( \overline{d} = \overline{d} \) and \( S_x = S_D \).

2 Proportions

\[ H_0 : p_1 = p_2 \]
\[ H_1 : p_1 \neq p_2 \]

Test statistic \( z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\hat{p}(1 - \hat{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \) (usually \( p_1 - p_2)_0 = 0 \) where \( \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} \), \( \hat{q} = 1 - \hat{p} \)

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [2-PropZTest] and press the [ENTER] key. Type in the X1, n1, X2, n2, arrow over to the ≠, <, > sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the z-test statistic and the p-value. Note X1 and X2 need to be a whole number, not a decimal.

Confidence Interval: \( (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}} \) where \( \hat{p}_1 = \frac{x_1}{n_1} \), \( \hat{p}_2 = \frac{x_2}{n_2} \).

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [7-2-PropZInterval] and press the [ENTER] key. Type in the X1, n1, X2, n2, the confidence level, then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval.

2 Variances or Standard Deviations

\[ H_0 : \sigma_1^2 = \sigma_2^2 \] or \[ H_0 : \sigma_1 = \sigma_2 \]
\[ H_1 : \sigma_1^2 \neq \sigma_2^2 \] or \[ H_1 : \sigma_1 \neq \sigma_2 \]

Both use test statistic \( F = \frac{s_1^2}{s_2^2} \) with dfN = n1 - 1 and dfD = n2 - 1.

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [2-SampFTest] and press the [ENTER] key. Arrow over to the [Stats] menu and press the [Enter] key. Then type in the s1, n1, s2, n2, arrow over to the ≠, <, > sign that is the same in the problems alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic F and the p-value.

Goodness of Fit Test
- “There is no preference” w/ 4 groups H0: p1=.25, p2=.25, p3=.25, p4=.25
- “There is a preference” H1: At least one proportion is different.

Source: Rachel L. Webb, PSU, January 2018
Learning Center 1875 SW Park Avenue, Millar Library, Portland, OR 97201 503.725.4448 www.pdx.edu/tutoring
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Proportions are 1/k or different percentages for each group given in the problem. If %’s are given use those decimals for each $p_i$. Expected values are found by taking each group’s proportion times the sample size ($p_i \times n$), $df=k-1$. Test statistic: $\chi^2 = \sum \frac{(O - E)^2}{E}$

- For the TI-84: Enter the observed frequencies in List1, the expected frequency in List2, press [STAT] key, arrow over to [TESTS] menu, arrow down to [$\chi^2$GOF-Test] (not available on the TI-83 and some TI-84 calculators)
- For the TI-89: Hypothesis test for three or more proportions (goodness of fit test). Go to the [Apps] Stat/List Editor, then type in the observed values into list 1, and the expected values into list 2. Select 2nd then F6 [Tests], then select 7: Chi-2GOF. Type in the list names and the degrees of freedom ($df = k-1$). Then press the [ENTER] key to calculate. The calculator returns the $\chi^2$-test statistic and the p-value.

**Test for Independence**

H0: Variable 1 is independent of Variable 2  
H1: Variable 1 is dependent of Variable 2

Expected values are found by taking $\frac{\text{(Row Total)} \times (\text{Column Total})}{\text{Grand Total}}$ for each cell.

Test statistic: $\chi^2 = \sum \frac{(O - E)^2}{E}$.

- For the TI-83 or TI-84: Press the [2nd] then [MATRIX] key. Arrow over to the EDIT menu and 1:[A] should be highlighted, press the [ENTER] key. For a m X n contingency table, type in the number of rows(m) and the number of columns(n) at the top of the screen so that it looks like this $\text{MATRIX}[A] m \times n$. For example a 2 x 3 contingency table, the top of the screen would look like this $\text{MATRIX}[A] 2 \times 4$, as you hit [ENTER] the table will automatically widen to the size you put in. Now enter all of the observed values in there proper positions. Then press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option $[\chi^2$-Test] and press the [ENTER] key. Leave the default as Observed:[A] and Expected:[B], arrow down to [Calculate] and press the [ENTER] key. The calculator returns the $\chi^2$-test statistic and the p-value. If you go back to the matrix menu [2nd] then [MATRIX] key, arrow over to EDIT and choose 2:[B], you will see all of the expected values.

- TI-89: First you need to create the matrix for the observed values: Press: [Home] to return to the Home screen, press [Apps] and select 6:Data/Matrix Editor. A menu is displayed, select 3:New. The New dialog box is displayed. Press the right arrow key to highlight 2:Matrix, and press [ENTER] to choose Matrix type. Press the down arrow key to highlight 1:main, and press [ENTER], to choose main folder. Press the down arrow key, and then enter the name o in the Variable field. Enter 3 for Row dimension and 2 for Column dimension. Press [ENTER] to display the matrix editor. Enter 4, 9, 5 in c1 and 7, 2, 3 in c2. Press  llam [Apps] [ENTER] to close the matrix editor and return to the list editor. If you have more than one Application loaded, press  llam [Apps], and then select Stats/List Editor.

To display the Chi-square 2-Way dialog box, press 2nd then F6 [Tests], then select 8: Chi-2 2-way. Enter in in the Observed Mat: o; Store Expected to: statvar; Store CompMat to: statvar. This will store the expected values in the matrix folder statvars with the name e, and the $(o-e)^2/e$ values in the matrix c. Press the [ENTER] key to calculate. The calculator returns the $\chi^2$-test statistic and the p-value. If you go back to the matrix menu you will see all of the expected and $(o-e)^2/e$ values.

**Correlation and Regression**

$H_0: \rho = 0$  
$H_1: \rho \neq 0$

Test statistic for correlation: $t = r \sqrt{\frac{(n-2)}{(1-r^2)}}$ with df= $n-2$.

- In the [STAT] editor enter the x values into list one and the y values into list two. In the [TESTS] menu, arrow down to the option [LinRegTTest] and press the [ENTER] key. The default is Xlist: L1, Ylist: L2, Freq: 1. $\beta$ and $\rho \neq 0$. Arrow down to Calculate and press the [ENTER] key. The calculator returns the t-test statistic, the y-intercept $a$, slope $b$, the standard error of estimate $s=s_{est}$, the coefficient of determination $R^2$, and the correlation coefficient $r$.
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Prediction Interval for a predicted value of y: \(\hat{y} \pm t_{a/2} \cdot s_{\text{se}} \sqrt{\frac{1}{n} + \frac{n(x-\bar{x})^2}{n \cdot \sum x^2 - (\sum x)^2}}\) with df = n - 2.

\[ R^2_{adj} = 1 - \left(1 - R^2 \left(\frac{n-1}{n-p-1}\right)\right) \]

One-Factor ANOVA table  \(k=\# \text{of groups}, N=\text{total of all } n's\)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F (Test Statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (Treatment or Factor)</td>
<td>(SS_B = \Sigma n(\bar{x} - \bar{\bar{x}}))</td>
<td>(k-1)</td>
<td>(MS_B = SS_B / df_B)</td>
<td>(F = MS_B / MS_W)</td>
</tr>
</tbody>
</table>
| Within (Error)          | \(SS_W = \Sigma(n-1)s^2\) | \(N-k\) | \(MS_W = SS_W / df_W\) | |}
| Total                   | \(SS_T\)    | \(N-1\) | | |}

\(H_0: \mu_1=\mu_2=\mu_3=...=\mu_k\)
\(H_1: \text{At least one mean is different}\)

CV: Always right-tailed, use \(df = N - 1\) and \(df = df_W\)

- For the TI-83 or TI-84: Note you have to have the actual raw data to do this test on the calculator. Press the [STAT] key and then the [EDIT] function, type the three lists of data into list one, two and three. Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [F:ANOVA] and press the [ENTER] key. This brings you back to the regular screen where you should now see ANOVA(). Now hit the \(2^{nd}\) [L_1] [,] \(2^{nd}\) [L_2] [,] \(2^{nd}\) [L_3] [)] keys in that order. You should now see ANOVA(\(L_1, L_2, L_3\)), if you had 4 lists you would then have an additional list. Press the [ENTER] key. The calculator returns the F-test statistic, the p-value, Factor (Between) \(df\), SS and MS. Error (Within) \(df\), SS and MS. The last value \(Sx\) is the square root of the MSE.

- For the TI-89: Go to the [Apps] Stat/List Editor, then type in the data for each group into a separate list, (if you do not have the raw data, enter the sample size, sample mean and sample variance for group 1 into list1 in that order, repeat for list2, etc.). Select 2nd [List] then F6 [Tests], then select C:ANOVA. Select the input method data or stats. Select the number of groups. Press the [ENTER] key to calculate. The calculator returns the F-test statistic, the p-value, Factor (Between) \(df\), SS and MS. Error (Within) \(df\), SS and MS. The last value \(Sx\) is the square root of the MSE.

When you reject \(H_0\) for a one-factor ANOVA then you should do a multiple comparison. For example for 3 groups you would have the following 3 comparisons. (4 groups would have \(C_2=6\) comparisons)

\(H_0: \mu_1=\mu_2\) \(H_0: \mu_1=\mu_3\) \(H_0: \mu_2=\mu_3\)
\(H_1: \mu_1\neq \mu_2\) \(H_1: \mu_1\neq \mu_3\) \(H_1: \mu_2\neq \mu_3\)

- Scheffé Test Statistic (unequal n): \(F_S = \frac{(\bar{x}_i - \bar{x}_j)^2}{MSE \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}\). Critical Valuc: \((k-1)\times(CV\text{ from original ANOVA})\)

- Bonferroni Test \(t = \sqrt{\frac{(\bar{x}_i - \bar{x}_j)^2}{MSE \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}\) with \(df = N - k\) and multiply the p-value by \(C_2\) groups. For example if you use the tcdf in your calculator to find the area in both the tails and you have 4 groups you would multiply the tail areas by \(C_2=6\).

Two-Factor ANOVA table \(a=\# \text{ of row factors}, b=\# \text{ of column factors}, n=\text{sample size in one group}, N=\text{total}\)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F (Test Statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row (Factor A)</td>
<td>(SS_A)</td>
<td>(a-1)</td>
<td>(MS_A = SS_A / df_A)</td>
<td>(F_A = MS_A / MS_E)</td>
</tr>
<tr>
<td>Column (Factor B)</td>
<td>(SS_B)</td>
<td>(b-1)</td>
<td>(MS_B = SS_B / df_B)</td>
<td>(F_B = MS_B / MS_E)</td>
</tr>
<tr>
<td>Interaction (A \times B)</td>
<td>(SS_{AB})</td>
<td>((a-1)(b-1))</td>
<td>(MS_{AB} = SS_{AB} / df_{AB})</td>
<td>(F_{AB} = MS_{AB} / MS_E)</td>
</tr>
</tbody>
</table>
| Error (Within)          | \(SS_E\)    | \(ab(n-1)\) | \(MS_E = SS_E / df_E\) | |}
| Total                   | \(SS_T\)    | \(N-1\) | | |}

Row Effect (Factor A): \(H_0: \text{The row variable has no effect on the average } \)
\(H_1: \text{The row variable has an effect on the average } \)