Common Errors in Algebra and Calculus

This is the first draft of a compilation of common arithmetic and conceptual mistakes in most, lower division math courses. These are conveyed with examples (rather than general forms) and brief explanations for each topic.

Algebra

0! ≠ 0 because n! = (n+1)!/(n+1) so 0! = (0+1)!/(0+1) = 1

1/0 ≠ 0. Observe the graph of f(x) = 1/x at x=0

x² • x³ ≠ x⁶ because x² • x³=(x•x)(x•x•x)=x⁵

√2 + √3 ≠ √5 (Sometimes you just cannot ‘simplify’)

(x + y)² ≠ x² + y² because (x + y)² = (x+y)(x+y)=(x+y)x + (x+y)y = x² + 2xy + y²

3(x + y)² ≠ (3x + 3y)² because 3(x + y)² = 3(x² + 2xy + y²) = 3x² + 6xy + 3y²

√16 ≠ ±4 because √16 = 2² = 2²/2 = 2² = 4. However, the solutions to x² = 16 are x=4 or x=-4

Similarly, -2² ≠ 4 but (-2)² = 4. Parentheses are important!

The equations 2x²=x is not the same as 2x=1 because 2x=1 does not have the same solutions as 2x²=x. This ‘reduction’ does not simplify the equation 2x²=x; it changes it.

Calculus

Lim x —>2 (x²-4)/(x-2) ≠ (x-2)(x+2)/(x-2), but Lim x —>2 (x²-4)/(x-2) = Lim x —>2 (x-2)(x+2)/(x-2) = 4. Cannot drop the limit between steps or else the statement does not make sense.

d/dx ln(x²) ≠ 1/x² because d/dx ln(x²) = d/dx 2 ln(x) = 2/x.

Also, ∫(1/x)dx = ln(x) does not imply ∫(1/x²)dx = ln(x²)

d/dx e^x ≠ xe^[x-1] because x is a variable; it is not fixed.

∫(e^x)dx ≠ e^[x] but rather 4e^[x]

∫(√(x³))dx = (2/3)x^(3/2) does not imply ∫(√(x² + 1))dx = (2/3)(x² + 1)³/2

Using l'Hospital’s Rule: It states, “Let lim stand for the limit lim x —> c , lim x —> c⁻, lim x —> c⁺, lim x —> ±∞, or lim x —> -∞), and suppose that lim f(x) and lim g(x) are both zero or are both ± ∞. If

lim(f(x))/(g(x)) has a finite value or if the limit is +/-inf, then lim(f(x))/(g(x))=lim(f(x))/(g(x)).” This means one cannot simply state lim(f(x))/(g(x))=lim(f(x))/(g(x)); all the premises must be stated before the conclusion

∫x+2 is ambiguous; this could be interpreted as ∫xdx +2. ∫(x+2)dx is much more clear. And remember “+C” at the end of the solution!

∫Sin(α)dx does not make sense. The variables must match!

Finally, “Euler” is pronounced “Oy-ler” not “You-ler”.

Source: LC Tutor, Nathan Lawrence, spring 2015