Common Errors in Algebra and Calculus

This is the first draft of a compilation of common arithmetic and conceptual mistakes in most, lower division math courses. These are conveyed with examples (rather than general forms) and brief explanations for each topic.

Algebra

$0! \neq 0$ because $n! = (n+1)!/(n+1)$ so $0! = (0+1)!/(0+1) = 1$

$1/0 \neq 0$. Observe the graph of $f(x) = 1/x$ at $x=0$

$x^2 \bullet x^3 \neq x^6$ because $x^2 \bullet x^3=(x\bullet x)(x\bullet x\bullet x)=x^5$

$\sqrt{2} + \sqrt{3} \neq \sqrt{5}$ (Sometimes you just cannot 'simplify')

$(x + y)^2 \neq x^2 + y^2$ because $(x + y)^2 = (x+y)(x+y)=(x+y)x + (x+y)y = x^2 + 2xy + y^2$

$3(x + y)^2 \neq (3x + 3y)^2$ because $3(x + y)^2 = 3(x^2 + 2xy + y^2) = 3x^2 + 6xy + 3y^2$

$\sqrt{16} \neq \pm 4$ because $\sqrt{16} = \sqrt{2^4} = 2^{4/2} = 2^2 = 4$. However, the solutions to $x^2 = 16$ are $x=4$ or $x= -4$

Similarly, $-2^2 \neq 4$ but $(-2)^2 = 4$. Parentheses are important!

The equations $2x^2=x$ is not the same as $2x=1$ because $2x=1$ does not have the same solutions as $2x^2=x$. This 'reduction' does not simplify the equation $2x^2=x$; it changes it.

Calculus

Lim $x \longrightarrow 2$ $(x^2-4)/(x-2) \neq (x-2)(x+2)/(x-2)$, but Lim $x \longrightarrow 2$ $(x^2-4)/(x-2) =$ Lim $x \longrightarrow 2$ $(x-2)(x+2)/(x-2) = 4$. Cannot drop the limit between steps or else the statement does not make sense.

$d/dx \ln(x^2) \neq 1/x^2$ because $d/dx \ln(x^2) = d/dx\ 2 \ln(x) = 2/x$.

Also, $\int(1/x)dx = \ln(x)$ does not imply $\int(1/x^2)dx = \ln(x^2)$

$d/dx\ e^x \neq xe^{(x-1)}$ because x is a variable; it is not fixed.

$d/dx\ e^{4x} \neq e^{4x}$ but rather $4e^{4x}$

$\int(\sqrt{x})dx = (2/3)x^{3/2}$ does not imply $\int(\sqrt{(x^2 + 1)})dx = (2/3)(x^2 + 1)^{3/2}$

Using <u>l'Hospital's Rule</u>: It states, "Let lim stand for the limit lim $x \longrightarrow c$, lim $x \longrightarrow c^-$, lim $x \longrightarrow c^+$, lim $x \longrightarrow \infty$), or lim $x \longrightarrow -\infty$), and suppose that lim f(x) and lim g(x) are both zero or are both $\pm\infty$. **If** lim(f'(x))/(g'(x)) has a finite value or **if** the limit is +/-infty, **then** lim(f(x))/(g(x))=lim(f'(x))/(g'(x)) ." This means one cannot simply state lim(f(x))/(g(x))=lim(f'(x))/(g'(x)); all the premises must be stated before the conclusion

$\int x+2$ is ambiguous; this could be interpreted as $\int x dx +2$. $\int(x+2)dx$ is much more clear. And remember "+C" at the end of the solution!

$\int \text{Sin}(\o)dx$ does not make sense. The variables must match!

Finally, "Euler" is pronounced "Oy-ler" not "You-ler".

Source: LC Tutor, Nathan Lawrence, spring 2015