Mechanics

1. A spring-loaded gun is mounted on a block and loaded with a marble. A target box is on the floor and located at a distance $D = 2.20 \text{ m}$ from the edge of the block, as shown in the figure.

If the spring is compressed 1.10 cm, the marble falls short of the box by 27.0 cm. How far should the spring be compressed such that the marble hits the box? Assume neither the spring nor the marble encounters friction.

(10 points)

2. When two hydrogen atoms of mass $m$ combine to form a diatomic hydrogen molecule $\text{H}_2$, the potential energy of the system after they combine is $-\Delta$, where $\Delta$ is a positive quantity called the binding energy of the molecule.

2.A Show that in a collision that involves only two hydrogen atoms, it is impossible to form an $\text{H}_2$ molecule because momentum and energy cannot simultaneously be conserved. (Hint: use the center of mass reference frame) (5 points)

2.B An $\text{H}_2$ molecule can be formed in a collision that involves three hydrogen atoms. Suppose that before such a collision, each of the three atoms has speed 1000 m/s, and they are approaching at 120° angles so that at any
instant, the atoms lie at the corners of an equilateral triangle. Find the speeds of the $\text{H}_2$ molecule and of the single hydrogen atom that remains after the collision.

The binding energy of $\text{H}_2$ is $\Delta = 7.23 \times 10^{-19}$ J, and the mass of the hydrogen atom is $1.67 \times 10^{-27}$ Kg. 

(5 points)

3. Find the center of mass of a solid, uniform density, hemisphere of radius “$R$”. Calculation is required; do not just give the answer.

(10 points)

Helpful integrals

\[
\int \sin ax \cos^m ax \, dx = -\frac{\cos^{m+1} ax}{(m+1)a} \\
\int \cos ax \sin^m ax \, dx = -\frac{\sin^{m+1} ax}{(m+1)a}
\]

4. A rigid body is made of three identical thin rods, each of length $L=0.60$ m, fasten together in the form of a letter “H”. The structure is free to rotate about a horizontal axis (axis-Y in the figure.)
Starting from a position in which the plane of the “H” is horizontal (XY plane), the body is allowed to fall from rest. What is the angular speed of the body when the plane of the H is vertical?

Hint: The moment of inertia of a thin rod rotating about an axis passing through its end and perpendicular to its length is \((1/3)ML^2\), where \(M\) is its mass and \(L\) is its length.

(10 points)
Thermodynamics

1. Consider the following experiment to take place in a closed system. A 10g block of solid copper at a temperature of 77°C is placed in thermal contact with a 20g quantity of solid copper at a temperature of 17°C. The specific heat \((c_v)\) of metallic copper is 0.389 J g\(^{-1}\) °C\(^{-1}\) over this temperature range.

1A Find the quantity of energy transferred \((\Delta U)\) between the two metallic copper solids as the combined system comes to thermal equilibrium. Also calculate the final temperature \((T_f)\) of the two metal blocks.

(5 points)

1B Calculate the total change in entropy \((\Delta S)\) in this exchange of heat between the two bodies. Is this in agreement with the Second Law of thermodynamics?

(5 points)

Constants and Unit Conversions

- \(R = 8.315 \text{ J/(mol K)}\)
- \(k = 1.38 \times 10^{-23} \text{ J/K}\)
- 1 cal = 4.186 J

2. Consider nitrogen gas \((\text{N}_2)\) in a cylinder with a frictionless piston. Initially the cylinder and gas are at room temperature (25°C). It expands so that the volume and pressure rise proportionally from \(V_i = 1 \text{ L}\) and \(P_i = 1 \text{ atm}\) to \(V_f = 3 \text{ L}\) and \(P_f = 3 \text{ atm}\). Assume \(\text{N}_2\) is an ideal gas and has 5 degrees of freedom (3 translation and 2 rotation) over this temperature range.

2A Calculate the work done \((W)\) on the nitrogen gas in the cylinder during this expansion.

(4 points)
2B  Calculate the change in energy ($\Delta U$) of the nitrogen gas inside the cylinder during this transformation.
   (4 points)

2C  Calculate the heat added (Q) to the nitrogen gas in order for the process to occur.
   (2 points)

**Constants and Unit Conversions**

- $k = 1.38 \times 10^{-23}$ J/K
- $R = 8.315$ J/molK
- $1 \text{ atm} = 760 \text{ mmHg} = 1.013 \text{ bar} = 1.013 \times 10^5 \text{ N/m}^2$
- $1 \text{ L} = 10^{-3} \text{ m}^3$
- 1 mol ideal gas at 25°C and 1 atm occupies 24.5L

---

3. The figure shows two compartments separated by a thin wall. The left side contains 0.060 mol of helium at an initial temperature of 600 K and the right side contains 0.030 mol of helium at an initial temperature of 300 K. The compartment on the right is attached to a vertical cylinder. A 10 cm diameter, 2.0 kg piston can slide without friction up and down the cylinder. The compartments are thermally isolated from the outside. The thin wall is stationary and thermal conductive. The volumes of the compartments are unknown. The exterior pressure is 1.0 atm.

3A  What is the final temperature?  (2.5 points)
3B How much heat is transferred from the left side to the right side? (2.5 points)

3C How high is the piston lifted due to this heat transfer? (2.5 points)

3D What fraction of the heat is converted into work? (2.5 points)

4. A heat engine using 5.00 g of helium gas is initially at STP (atomic mass of He is 4 g/mol.) The gas goes through the following closed cycle:
- Isothermal compression until the volume is halved.
- Isobaric expansion until the volume is restored to its initial value.
- Isochoric cooling until the pressure is restored to its initial value.

4A. How much work does this engine do per cycle? (5 points)

4B. What is the thermal efficiency? (5 points)