1. An electron is in a state given by the wave function:

\[ \psi(x) = Ae^{-\frac{(x-b)^2}{2\sigma^2}} \]

(a.) Determine the value of A.

(b.) Determine the expectation value (\(\bar{x}\)) of the position.

(c.) Determine the Uncertainty (\(\Delta x\)) in position.
2. The Schrödinger Equation for a quantum oscillator is:

\[ \frac{\partial^2 \psi}{\partial x^2} = \frac{2m}{\hbar} \left( \frac{m\omega^2 x^2}{\hbar} - E \right) \psi \]

The wavefunction for the ground state is: \[ \psi(x) = Ce^{-2\alpha x^2} \]

Show that \[ \psi(x) = Ce^{-2\alpha x^2} \] satisfies the Schrödinger Equation for this system (by showing that \( \alpha \) is a function of particle mass \( m \), angular frequency \( \omega \) and Planck's constant) and find the energy of this state.
3. Describe how you would use the photoelectric effect and a series of measurements of photoelectron energy to determine the value of Planck’s constant.
4. The Maxwell speed distribution for gas molecules at thermal equilibrium at temperature $T$ is given by:

$$n(v) = \frac{4\pi N}{V} \left( \frac{m}{2\pi k_B T} \right)^{3/2} v^2 \exp \left( \frac{-mv^2}{2k_B T} \right)$$

Find an expression for the most probable speed of a gas molecule.
5. A solid at temperature $T$ can be viewed as a system of quantized harmonic oscillators with discrete energy levels separated by $\hbar \omega$. The oscillators can absorb thermal energy only if the temperature is high enough that the average thermal energy of the oscillator, $\bar{E}$, is approximately equal to the oscillator energy-level spacing, $\hbar \omega$. For low temperatures such that $\bar{E} < \hbar \omega$, there is so little thermal energy available that the atoms cannot even be raised to the first excited state and the specific heat tends to zero. **Show** that the carbon atoms in diamond are effectively decoupled from the thermal energy at room temperature but can absorb energy at a temperature of 1500 K.