4.1 Introduction
4.2 TM radiation: Electric field parallel to the plane of incidence
   4.2.A General description of TM fields
   4.2.B Special case: Evanescently confined electromagnetic TM-waves
       EM waves in the dielectric medium, EM waves in the metallic medium
       relationship between $\bar{E}$ and $\bar{B}$ in the same medium
   4.2.C Boundary conditions: Relationship between the fields across the interface
       Continuity of $\bar{B}/\mu$, continuity of $\bar{E}$.
4.3 Dispersion relationship of TM surface-plasmon-polariton waves propagating along
   a metal-insulator interface
   4.3.A General case
   4.3.B Case: Ideal conductor $\text{Im}(\varepsilon_m) = 0$
       Surface plasmon frequency
   4.3.C Case: conductor with $\text{Im}(\varepsilon_m) \neq 0$
   4.3.D Effects of different dielectrics
4.4 Excitation of SPP through prism coupling
   4.4.A The Otto configuration
   4.4.B The Kretschmann configuration
4.5 Bio-applications of SPP
4.6 SPP and Quantum Entanglement: The quantum nature of SPP
The case for Plasmons

Promising technology
Just over a decade ago, the term “plasmonics” was coined for a promising new device technology that aims to exploit the unique optical properties of metallic nanostructures to enable routing and active manipulation of light at the nanoscale (1).

At the same time, it was already well established that tiny metallic particles have a number of valuable optical properties that are derived from their ability to support collective light-induced electronic excitations, known as surface plasmons. Most notably, nanostructured metals dramatically alter the way light scatters from molecules, and this later led to the development of an important optical spectroscopy technique called surface-enhanced Raman spectroscopy (2–4).

The plasmonics field exploded when it was demonstrated that
- metallic nanowires enable much smaller optical circuitry than dielectric (e.g. glass) waveguides (5),
- metal films with nanoscale holes can show extraordinarily high optical transmission (6), and
- a simple thin film of metal can serve as an optical lens (7).

Plasmonic elements are also important as popular components of metamaterials—artificial optical materials with rationally designed geometries and arrangements of nanoscale building blocks. The burgeoning field of transformation optics elegantly describes how such materials can facilitate an unprecedented control over light by “engineering optical space” with local control of a metamaterial’s response (8).

Challenges
As these novel phenomena captured the imagination of a broad audience, some of the severe limitations of metals were being recognized. The most important challenge is that metals exhibit substantial resistive heating losses when they interact with light. We have an opportunity to look back, evaluate the progress in the field, and look at promising future directions.

Applications
A diverse set of plasmonics applications has emerged in the past 10 years. Early applications included the development of high performance near-field optical microscopy and biosensing methods.

More recently, many new technologies have emerged in which the use of plasmonics seems promising, including
- thermally assisted magnetic recording (9),
- thermal cancer treatment (10),
- catalysis and nanostructure growth (11), and
- computer chips (12, 13).
Interestingly, the first three applications in this list capitalize on light-induced heating, which was originally considered as a weakness of plasmonics.

After the discovery that long-distance information transport on chips with plasmonic waveguides would suffer too strongly from heating effects (14), it now has been established that modulators (12) and detectors (13) can be realized that meet the stringent power, speed, and materials requirements necessary to incorporate plasmonics into conventional electronics technology.

Plasmonic sources capable of efficiently coupling quantum emitters to a single, well defined optical mode may first find applications in the field of quantum plasmonics and later in power-efficient chip-scale sources (15, 16). In this respect, the recent prediction (17) and realization (18– 20) of coherent nanometallic light sources constitutes an extremely important development. Looking even further down the line, the recent prediction that a surface plasmon laser may operate as an ultrafast amplifier is certainly stimulating. Could one build ultrafast logic circuits with devices that perform a similar function as the ubiquitous transistor but are orders of magnitude faster (21)?

**Plasmons as an alternative to complement semiconductor devices**

It has become clear what role plasmonics can play in future device technologies and how it can complement electronics and conventional photonics (see the figure). Each of these device technologies can perform unique functions that play to the strength of the key materials.

- The electrical properties of semiconductors enable the realization of truly nanoscale elements for computation and information storage. The high transparency of dielectrics facilitates information transport over long distances and at incredible data rates.
- Unfortunately, semiconductor electronics is limited in speed by interconnect delay-time issues, and dielectric photonics is limited in size by the fundamental laws of diffraction.
- Plasmonics offers the opportunity to combine the size of nanoelectronics and the speed of dielectric photonics, enabling devices that might naturally interface with similar-speed photonic devices and with similar-size electronic components, thus enhancing the synergy between these technologies. The semiconductor and photonics industries have continued to develop rapidly, and it will be exciting to see what the next decade will bring for plasmonics.

**References**

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4.1 Introduction

**Elementary excitations:**
- Phonons
- Plasmons
- Exciton (bound electron-hole pair)

**Polaritons**
Coupled state between an elementary excitation and a photon.

**Plasmon polariton**
Coupled state between a plasmon and a photon.

**Phonon polariton:** coupled state between a phonon and a photon.

**Surface plasmon polaritons** are electromagnetic excitations propagating along a metal/dielectric (XY) interface, and evanescently confined in the direction (Z) perpendicular to the interface.

**Objective**

The objective of this lecture is to characterize surface plasmon polaritons $\sim e^{i(\beta x - \omega t)}$ (propagating in the XY plane) in terms of its dispersion relationship $\beta = \beta(\omega)$. 
4.2 Transverse Magnetic (TM) radiation:  
Magnetic field perpendicular to the plane of incidence  
Electric field parallel to the plane of incidence

4.2.A General description of TM fields  
We describe first the TM radiation in a general context.  
(Subsequently, in the following sections we will adapt the results to tackle the particular  
case of evanescent TM fields at a dielectric/metal interface.)

![Fig. 1 Radiation of TM-polarization.](image)

Whether in the dielectric or the metal medium, the fields of TM-polarization radiation  
have the following form,

$$\vec{E}(\vec{r}, t) = (E_x, 0, E_z)$$
$$\vec{B}(\vec{r}, t) = (0, B_y, 0)$$

(1)

We have three unknown quantities, $E_x$, $E_z$, and $B_y$.

Maxwell Equations applied to TM fields

In light of the equivalency between the dielectric susceptibility $\varepsilon$ and the electrical  
conductivity $\sigma$ (addressed in the previous chapter), the Maxwell Equations are written as,

$$\nabla \cdot \varepsilon \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0$$
$$\varepsilon_0 c^2 \nabla \times \vec{B} = \frac{\partial}{\partial t} \varepsilon \vec{E}$$

where $\varepsilon$ stands for either $\varepsilon_d$ or $\varepsilon_m$.

Let’s solve these equations for the case when the fields vary harmonically with frequency  
$\omega$ as $e^{i\omega t}$. In this case, the equations take the form,

$$\nabla \cdot \varepsilon \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$
\[ \nabla \times \mathbf{E} - i\omega \mathbf{B} = 0 \quad \quad \quad \varepsilon_0 c^2 \nabla \times \mathbf{B} = -i\omega \varepsilon \mathbf{E} \]

\[ \nabla \times \mathbf{E} - i\omega \mathbf{B} = 0 \quad \quad \quad (2) \]

\[ \nabla \times \mathbf{E} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & E_z \end{vmatrix} = \left( \frac{\partial}{\partial y} E_z \right) , \left( \frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z \right) , -\left( \frac{\partial}{\partial y} E_x \right) \]

\[ i\omega \mathbf{B} = (0, i\omega B_y, 0) . \]

This implies,

\[ \frac{\partial E_z}{\partial y} = 0 \quad \quad \quad \text{(no variation of } E_z \text{ along the } y\text{-axis)} \]

\[ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega B_y \quad \quad \quad (3) \]

\[ \frac{\partial}{\partial y} E_x = 0 \quad \quad \quad \text{(no variation of } E_x \text{ along the } y\text{-axis)} \]

Notice also that (3) implies \( B_y \) does not depend on the \( y \) coordinate.

\[ \varepsilon_0 c^2 \nabla \times \mathbf{B} = -i\omega \varepsilon \mathbf{E} \quad \quad \quad (4) \]

\[ \nabla \times \mathbf{B} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_y & 0 \end{vmatrix} = \left( -\frac{\partial B_y}{\partial z} \right) , 0 , \left( \frac{\partial B_y}{\partial x} \right) \]

\[ -i\omega \varepsilon \mathbf{E} = -i\omega \varepsilon (E_x, 0, E_z) \]

which gives,

\[ \varepsilon_0 c^2 \frac{\partial B_y}{\partial z} = i\omega \varepsilon E_x \quad \quad \quad (5) \]

\[ \varepsilon_0 c^2 \frac{\partial B_y}{\partial x} = -i\omega \varepsilon E_z \]

Notice in (5) that \( E_x \) and \( E_z \) would be determined if \( B_y \) were known.

(Hence, in expression (1) only \( B_y \) needs to be determined then.)
Let’s try then to obtain the equation to be satisfied by $B_y$.

Expression (5) implies,

$$-i \frac{\varepsilon_0 c^2}{\omega \varepsilon} \frac{\partial^2 B_y}{\partial z^2} = \frac{\partial E_x}{\partial z}$$

$$i \frac{\varepsilon_0 c^2}{\omega \varepsilon} \frac{\partial^2 B_y}{\partial x^2} = \frac{\partial E_z}{\partial x}$$

which, together with (3) gives,

$$\frac{\partial^2 B_y}{\partial z^2} + \frac{\partial^2 B_y}{\partial x^2} + \left( \frac{\varepsilon}{\varepsilon_0} \frac{\omega^2}{c^2} \right) B_y = 0 \quad (6)$$

Given the fact that $B_y$ does not depend on the $y$ coordinate (as indicated by expression (3) above), expression (6) can be placed in a more succinct form,

$$\nabla^2 B_y + \left( \frac{\varepsilon}{\varepsilon_0} \frac{\omega^2}{c^2} \right) B_y = 0 \quad (6)'$$

The results in (5) and (6) offer a very interesting characteristic of the TM electromagnetic waves, which is that,

The $E$-fields are determined once the transverse magnetic field $B_y$ is known.

**Summary: Propagation of TM electromagnetic waves**

Once the field $B_y$ that satisfies the equation

$$\nabla^2 B_y + \left( \frac{\varepsilon}{\varepsilon_0} \frac{\omega^2}{c^2} \right) B_y = 0,$$

is found, then the remaining fields $E_x$ and $E_z$ can be found directly from the corresponding partial derivatives of $B_y$,

$$E_x = -i \frac{\varepsilon_0 c^2}{\varepsilon} \frac{\partial B_y}{\partial z}$$

$$E_z = i \frac{\varepsilon_0 c^2}{\varepsilon} \frac{\partial B_y}{\partial x}$$

The results above are valid for the dielectric medium as well as the metallic medium. This leads us to a first brief summary illustrated in Fig. 2 below.
4.2.B Special case: Evanescently confined electromagnetic TM-waves

To solve \( \nabla^2 B_y + \frac{\varepsilon}{\varepsilon_0} \frac{\omega^2}{c^2} B_y = 0 \), let’s consider solutions of the form,

\[
B_y(\mathbf{r},t) = B_y(\mathbf{r}) \ e^{-i\omega t} = B_0 e^{i\mathbf{k} \cdot \mathbf{r}} e^{-i\omega t}
\]

where \( B_0 \) is a constant amplitude.

*We are interested in fields propagating along the XY surface but localized in the z-direction, i.e. decreasing in amplitude away from the interface.*

(A “sandwiched” electromagnetic field.)

Following a similar procedure used in the description of total internal reflection, we consider complex wave vectors \( \mathbf{k} \) for the dielectric and metallic media, respectively.

\[
\mathbf{k}_d = (k_{dx}, 0, i\gamma_d) \quad \text{(for the region } z > 0; \text{ dielectric)}
\]

\[
B_y^d(\mathbf{r}) = B_0^d e^{ik_{dx}x} e^{-\gamma_d z}
\]

\[
\mathbf{k}_m = (k_{mx}, 0, -i\gamma_m) \quad \text{(for the region } z < 0; \text{ metal)}
\]

\[
B_y^m(\mathbf{r}) = B_0^m e^{ik_{mx}x} e^{\gamma_m z}
\]

What are our unknowns now?
$B_d^o$, $k_d$, $\gamma_d$, and $B_m^o$, $k_m$, $\gamma_m$

$k_{dx}(\omega)$, $k_{mx}(\omega)$

**Imposing the kinematic conditions**

At the interface $XY$, the boundary relationships to be satisfied is,

\[ k_{dx} = k_{mx}. \]

Let’s call this common optical constant $\beta$. Thus,

**In the dielectric medium** (region $z > 0$)

\[ \vec{k}_d = (\beta, 0, i \gamma_d) \]

\[ B^d_y(\vec{r}) = B^o_y e^{i \beta z} e^{-\gamma_d z} \]

\[ \nabla^2 B^d_y + \left( \frac{\varepsilon_d}{\varepsilon_0} \frac{\omega^2}{c^2} \right) B^d_y = 0 \implies \vec{k}_d \cdot \vec{k}_d = \beta^2 - \gamma_d^2 = \frac{\varepsilon_d \omega^2}{\varepsilon_0 c^2} \]

\[ \text{The larger value of } \beta, \text{ the larger value of } \gamma_d \]

Hence, the larger the value of $\beta$, the more localized is the wave in the $z$-direction

**In the metal medium** (region $z < 0$),

\[ \vec{k}_m = (\beta, 0, -i \gamma_m) \]

\[ B^m_y(\vec{r}) = B^o_y e^{i \beta z} e^{\gamma_m z} \]

\[ \nabla^2 B^m_y + \left( \frac{\varepsilon_m}{\varepsilon_0} \frac{\omega^2}{c^2} \right) B^m_y = 0 \implies \vec{k}_m \cdot \vec{k}_m = \beta^2 - \gamma_m^2 = \frac{\varepsilon_m \omega^2}{\varepsilon_0 c^2} \]

\[ \text{The larger value of } \beta, \text{ the larger value of } \gamma_m \]

Value determined by the material

\[ \text{Value determined by the material} \]
Fig. 3 Upon imposing the kinematic conditions (Snell’s law) the figure shows the evanescent TM fields at either side of the dielectric \((d) – \) metal \((m)\) interface. The real components of the \(k\) vectors are indicated, respectively, with blue and red arrows.

### 4.2.C Boundary conditions: Relationship between the fields across the interface

\[
\begin{align*}
\frac{\vec{B}}{\mu} & \quad \text{Tangential component of } \frac{\vec{B}}{\mu} \text{ at one side of the interface } z=0 \\
= & \quad \text{Tangential component of } \frac{\vec{B}}{\mu} \text{ at the other side of the interface } z=0 \\
\end{align*}
\]

**Continuity of \(\vec{B}/\mu\):**

We assume the magnetic permeability \(\mu\) is the same in both media.

From (8) and (8)', the evaluation at \(z = 0\) and the condition imposed by (9) implies,

\[
B_d^o = B_m^o = B_o
\]

**Continuity of \(\vec{E}\):**

The horizontal component of \(\vec{E}^d\) is \(E_x^d\), and according to (7)

\[
E_x = -i \frac{\varepsilon_0 c^2}{\varepsilon_d \omega} \frac{\partial B^d}{\partial z},
\]

Using the expression for \(B_y^d\) given in (8) and (10),

\[
\begin{align*}
E_x &= -i \frac{\varepsilon_0 c^2}{\varepsilon_d \omega} \frac{\partial}{\partial z} \left( B_o e^{i\beta x} e^{-\gamma_d z} \right) \\
&= i\gamma_d \frac{\varepsilon_0 c^2}{\varepsilon_d \omega} B_o e^{i\beta x} e^{-\gamma_d z}
\end{align*}
\]
Similarly, using the expression for $B^m_y$ given in (8)' and (10),

$$E^m_x = -i\gamma^m \frac{\varepsilon_0 c^2}{\varepsilon_m \omega} B^m_o e^{i\beta x} e^{i\gamma^m z}$$

Evaluating $E^d_x$ and $E^m_x$ at $z=0$, and imposing the boundary condition (9), gives,

$$i\gamma^d \frac{\varepsilon_0 c^2}{\varepsilon_d \omega} B^d_o e^{i\beta x} = -i\gamma^m \frac{\varepsilon_0 c^2}{\varepsilon_m \omega} B^m_o e^{i\beta x}$$

which has to be valid at all point on the XY interface. The latter requires $\frac{\gamma^d}{\varepsilon_d} = -\frac{\gamma^m}{\varepsilon_m}$, or equivalently,

$$\frac{\gamma^d}{\gamma^m} = -\frac{\varepsilon_d}{\varepsilon_m}$$  \hspace{1cm} (11)

**Condition for having evanescently confined waves**

![Diagram](image)

**Fig. 4** Surface plasmon polaritons wave propagating along a dielectric-metal interface ($\varepsilon_d$ and $\varepsilon_m$ are given), with a propagation wave-vector $\beta$ (to be determined). The figure specifies the magnetic field $B^d_y$; the corresponding electric fields can determined through expression (7). Notice the requirement of $\gamma^d / \gamma^m$ in terms of $\varepsilon_d / \varepsilon_m$.

To touch basis on the requirement stated in (11), the simplest case to consider SPP at a flat interface that is a non-absorbing medium with positive dielectric constant $\varepsilon_d$. Since we have setup the problem such that $\gamma^d$ and $\gamma^m$ are positive numbers (or at least that their real part is predominant,) expression (11) indicates that the conducting medium would have to have a negative dielectric constant $\varepsilon_m$. (The case in which $\varepsilon_m$ and, consequently, $\gamma^m$ are complex will be considered later). A real negative permittivity is obtained when a metal, modeled as Drude metal, has a negligible damping coefficient. Indeed, a Drude metal displays the following permittivity,
\[ \varepsilon(\omega) = \frac{\varepsilon_m(\omega)}{\varepsilon_0} = 1 - \frac{\omega_p^2}{(\omega^2 + \gamma_o^2)} + i \frac{\omega_p^2}{(\omega^2 + \gamma_o^2)} \frac{\gamma_o}{\omega}, \]

In the case that the damping part is negligible,

\[ \varepsilon_m(\omega) = \left[ 1 - \frac{\omega_p^2}{\omega^2} \right] \]

where the plasma frequency is given by,

\[ \omega_p^2 \equiv N e^2/\varepsilon_0 m \]

\( \varepsilon_m \) is negative as far as the frequency \( \omega \) is lower than the plasma frequency.

So far we have obtained \( \gamma_d/\gamma_m \) in terms of \( \varepsilon_d/\varepsilon_m \). We need to find \( \beta \) in terms of \( \varepsilon_d/\varepsilon_m \).

4.3 Dispersion relationship of waves propagating along a metal-insulator interface

4.3.A General case

Expressions (8) and (8)' established that,

\[ \gamma_d^2 = \beta^2 - \frac{\omega_p^2}{c^2} \frac{\varepsilon_d}{\varepsilon_0} \]
\[
\gamma_m^2 = \beta^2 - \frac{\omega^2}{c^2} \frac{E_m}{E_0}
\]

which leads to,

\[
\frac{\gamma'^2}{\gamma_m^2} = \frac{\beta^2 - \frac{\omega^2}{c^2} \frac{E_d}{E_0}}{\beta^2 - \frac{\omega^2}{c^2} \frac{E_m}{E_0}}
\]

Using \( \frac{\gamma_d}{\gamma_m} = \frac{E_d}{E_m} \), one obtains,

\[
\frac{(E_d/E_0)^2}{(E_m/E_0)^2} = \frac{\beta^2 - \frac{\omega^2}{c^2} \frac{E_d}{E_0}}{\beta^2 - \frac{\omega^2}{c^2} \frac{E_m}{E_0}}
\]

\[
\left[ \frac{(E_d/E_0)^2}{(E_m/E_0)^2} - 1 \right] \beta^2 = \frac{\omega^2}{c^2} \frac{(E_d/E_0)^2}{(E_m/E_0)^2} - \frac{\omega^2}{c^2} \frac{E_d}{E_0}
\]

\[
\left[ \frac{(E_d/E_0)^2}{(E_m/E_0)^2} - 1 \right] \beta^2 = \frac{\omega^2}{c^2} \left[ \frac{(E_d/E_0)^2}{(E_m/E_0)^2} - \frac{E_d}{E_0} \right]
\]

\[
\left[ \frac{(E_d/E_0)^2}{(E_m/E_0)^2} - 1 \right] \beta^2 = \frac{\omega^2}{c^2} \frac{E_d}{E_0} \left[ \frac{(E_d/E_0)}{(E_m/E_0)} - 1 \right]
\]

\[
\left[ \frac{(E_d/E_0)}{(E_m/E_0)} + 1 \right] \beta^2 = \frac{\omega^2}{c^2} \frac{E_d}{E_0}
\]

\[
\left[ \frac{(E_d/E_0) + (E_m/E_0)}{(E_m/E_0)} \right] \beta^2 = \frac{\omega^2}{c^2} \frac{E_d}{E_0}
\]

\[
\beta^2 = \frac{\omega^2}{c^2} \frac{(E_d/E_0) (E_m/E_0)}{(E_d/E_0) + (E_m/E_0)}
\]
\[ \beta = \frac{\omega}{c} \sqrt{\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)}} \quad \text{SPP dispersion relationship} \quad (12) \]

**Fig. 6** Given the permittivity of the dielectric medium \( \varepsilon_d \) and the conducting medium \( \varepsilon_m(\omega) \), the diagrams above describe the propagation (in the \( x \)-direction and with wave-vector \( \beta \)) of TM plasmon polariton waves (evanescent in the \( z \)-direction) formed at the flat XY-interface. Notice the requirement of \( \gamma_d / \gamma_m \) and \( \beta \) in terms of \( \varepsilon_d / \varepsilon_m \).

### 4.3.B Case: Ideal conductor \( \text{Im}(\varepsilon_m) = 0 \)

Consider the case of a conductive medium of negligible damping constant (free electrons gas),

\[
\frac{\varepsilon_m(\omega)}{\varepsilon_0} = \left[ 1 - \frac{\omega_p^2}{\omega^2} \right] \quad (13)
\]
To figure out the graph of $\omega$ vs $\beta$ corresponding to expression (12), it is simpler to analyze first the limit values that $\beta$ takes for different key values of $\omega$:

\[
\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)} = \frac{\varepsilon_d / \varepsilon_0}{1 - \frac{\omega_p^2}{\omega^2}}
\]

\[
(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0) = \left( \frac{\varepsilon_d}{\varepsilon_0} + 1 - \frac{\omega_p^2}{\omega^2} \right)
\]

\[
\beta = \frac{\omega}{c} \sqrt{\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)}}
\]

\[
\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)} \rightarrow \frac{\varepsilon_d}{\varepsilon_0} \quad \text{and} \quad \beta \rightarrow \frac{\omega}{c} \sqrt{\frac{\varepsilon_d}{\varepsilon_0}}
\]

That is, for small $\omega$

\[
\omega = v_d \beta
\]

where $v_d$ is the speed of light in the dielectric medium.

- Notice that the denominator in the expression for $\beta$,

\[
(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0) = \left( \frac{\varepsilon_d}{\varepsilon_0} + 1 - \frac{\omega_p^2}{\omega^2} \right)
\]

will vanish when $\omega$ fulfills

\[
\varepsilon_d / \varepsilon_0 + 1 = \omega r^2 / \omega^2;
\]
That is, the denominator in the expression for $\beta$ vanishes when

$$\omega = \omega_p / \sqrt{(\epsilon_d / \epsilon_0) + 1}.$$ 

Accordingly,

\[
\frac{(\epsilon_d / \epsilon_0)(\epsilon_m / \epsilon_0)}{(\epsilon_d / \epsilon_0) + (\epsilon_m / \epsilon_0)} \rightarrow \omega \rightarrow \frac{\omega_p}{\sqrt{(\epsilon_d / \epsilon_0) + 1}} \rightarrow \infty
\]

and

\[
\beta \rightarrow \frac{\omega_p}{\sqrt{(\epsilon_d / \epsilon_0) + 1}} \rightarrow \infty
\]

\[
\omega(\beta)
\]

\[
\omega_p
\]

\[
\omega_p \sqrt{(\epsilon_d / \epsilon_0) + 1}
\]

\[
\beta vs \omega
\]

for large $\omega$

\[
\beta vs \omega
\]

for small $\omega$

\[
0
\]

\[
\beta
\]

Corrected text

- When $\omega$ is a bit greater than $\frac{\omega_p}{\sqrt{(\epsilon_d / \epsilon_0) + 1}}$, but lower than $\omega_p$

the denominator $(\epsilon_d / \epsilon_0) + (\epsilon_m / \epsilon_0) = \left( \frac{\epsilon_d}{\epsilon_0} + 1 - \frac{\omega_p^2}{\omega^2} \right)$ will be positive

(but close to zero), while

the numerator $(\epsilon_d / \epsilon_0)(\epsilon_m / \epsilon_0) = \left( \frac{\epsilon_d}{\epsilon_0} \left( 1 - \frac{\omega_p^2}{\omega^2} \right) \right)$ is a negative.

Hence,
\[
\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)} < 0 \quad \text{and} \quad \beta = \frac{\omega}{c} \sqrt{\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)}}
\]

\(\beta\) is a complex number

i.e. propagation of SPP is forbidden in the frequency range of \(\frac{\omega_p}{\sqrt{(\varepsilon_d / \varepsilon_0) + 1}} < \omega < \omega_p\)

\[
\frac{\varepsilon_d}{\varepsilon_0} \left( 1 - \frac{\omega_p^2}{\omega^2} \right)
\]

Notice the numerator \((\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0) = \frac{\varepsilon_d}{\varepsilon_0} \left( 1 - \frac{\omega_p^2}{\omega^2} \right)\) cancels out when \(\omega = \omega_p\).

The last two results are summarized below

- In the forbidden frequency range, \(\frac{\omega_p}{\sqrt{(\varepsilon_d / \varepsilon_0) + 1}} < \omega < \omega_p\),
  \(\beta\) has the following behavior,
  \[
  \frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)} \xrightarrow[\omega \to \omega_p]{} 0 \quad \text{and} \quad \beta \xrightarrow[\omega \to \omega_p]{} 0i
  \]
  \[
  \frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)} \xrightarrow[\omega \to \omega_p / \sqrt{(\varepsilon_d / \varepsilon_0) + 1}]{} \infty i \quad \text{and} \quad \beta \xrightarrow[\omega \to \omega_p / \sqrt{(\varepsilon_d / \varepsilon_0) + 1}]{} \infty i
  \]
Let's address the confinement of the electromagnetic wave along the z-direction:

**CASE: For small wave-vectors \( \beta \)**

(Which corresponds to low frequencies in the mid-infrared or lower):

- \( \beta (\omega) \) is close to the light line \( k_d = (1/c_v)\omega \)
  where \( v_d \) is the speed of light in the dielectric medium and \( k_d = \frac{\omega}{c} \sqrt{\frac{\varepsilon_d}{\varepsilon_o}} \).

- For a given frequency \( \omega \), the value of \( \beta \) is actually a bit greater than the corresponding value of \( k_d \).

Therefore \( \gamma_d^2 = \beta^2 - \frac{\omega^2 \varepsilon_d}{c^2 \varepsilon_o} = \beta^2 - k_d^2 \) is a small fraction of \( k_d^2 \).

\( \gamma_d \) is a small fraction of \( k_d = \frac{2\pi}{\lambda_d} = \frac{2\pi}{\lambda/n_d} \)
so the waves \( B_y^d(\hat{r}) = B_c^y e^{i \beta x} e^{-\gamma_d z} \) decreases with \( z \), but a decay to \( e^{-1} \) occurs for \( z \)-values that extend many wavelengths (\( \lambda \)) into the dielectric medium.

\[ \gamma_m^2 = \beta^2 - \frac{\omega^2 \varepsilon_m}{c^2 \varepsilon_o} = \frac{\omega^2 \varepsilon_d}{c^2 \varepsilon_o} - \frac{\omega^2 \varepsilon_m}{c^2 \varepsilon_o} = \]
\[ = \frac{\omega^2}{c^2} \left[ \frac{\varepsilon_d}{\varepsilon_0} - 1 + \frac{\omega_p^2}{\omega^2} \right] \approx \frac{\omega_p^2}{c^2} \]
$$\gamma_m^2 \approx \frac{\omega_p^2}{c^2} \gg \frac{\omega^2}{c^2} = \left(\frac{2\pi}{\lambda}\right)^2$$

$$\gamma_m \gg \frac{2\pi}{\lambda}$$

Hence, $\gamma_m$ is very large, so the wave $B_y^m(\vec{r}) = B_o^m e^{i\beta z} e^{\gamma_m z^2}$ extends into the metal region a very small fraction of a wavelength ($\lambda$) distance.

These conclusions are verified by the results shown in Fig. 8.

![Fig. 8 Distribution of the magnetic field intensity for a surface plasmon wave (SPW) at the interface between gold and dielectric (refractive index of the dielectric is $n=1.32$). Ref: Jiří Homola, Present and future of surface plasmon resonance biosensors, Anal Bioanal Chem 377, 528 (2003).](image)

**CASE:** For very large wave-vectors $\beta$

Notice that $\beta$ approaches $\infty$ as $\omega$ approaches the value $\omega = \frac{\omega_p}{\sqrt{(\varepsilon_d / \varepsilon_0) + 1}}$.

Since $\gamma_d^2 = \beta^2 - \frac{\omega^2}{c^2} \frac{\varepsilon_d}{\varepsilon_0}$ and $\gamma_m^2 = \beta^2 - \frac{\omega^2}{c^2} \frac{\varepsilon_m}{\varepsilon_0}$, then an infinite value of $\beta$ implies that both $\gamma_d$ and $\gamma_m$ are very large. Hence, the wave practically remains literally constrained to the interface only.

Such a mode where $\beta \to \infty$ and $\omega = \omega_p / \sqrt{(\varepsilon_d / \varepsilon_0) + 1}$ is called surface plasmon.

Given its quite localized surface character this frequency mode is called surface plasmon frequency.

$$\omega_{SP} \equiv \frac{\omega_p}{\sqrt{(\varepsilon_d / \varepsilon_0) + 1}} \quad \text{surface plasmon frequency}$$ (14)
### 4.3.B Case: conductor with $\text{Im}(\varepsilon_m) \neq 0$

A real metal suffers from electron scattering as well as interband absorption, which leads to $\text{Im}(\varepsilon_m) \neq 0$.

A complex $\varepsilon_m$ implies that,

$$\beta = \frac{\omega}{c} \sqrt{\frac{(\varepsilon_d / \varepsilon_0)(\varepsilon_m / \varepsilon_0)}{(\varepsilon_d / \varepsilon_0) + (\varepsilon_m / \varepsilon_0)}}$$

is complex.

$$\gamma_d^2 = \beta^2 - \frac{\omega^2}{c^2} \frac{\varepsilon_d}{\varepsilon_0}$$

and

$$\gamma_m^2 = \beta^2 - \frac{\omega^2}{c^2} \frac{\varepsilon_m}{\varepsilon_0}$$

are also complex.

![Dispersion curve for SPP at the interface between a dielectric of positive permittivity $\varepsilon_d$ and a metal with $\text{Im}(\varepsilon_m) \neq 0$.](image)

**Fig. 9** Dispersion curve for SPP at the interface between a dielectric of positive permittivity $\varepsilon_d$ and a metal with $\text{Im}(\varepsilon_m) \neq 0$.

The consequence of a complex $\beta$ on the wave $B_{y}^{d,m}(\vec{r},t) = B_{m}^{d} e^{i\beta x} e^{-\gamma_{d,m}^2 x} e^{j\omega t}$ is that it becomes attenuated also in the $x$ direction,

$$e^{i\beta x} = e^{j(\text{Real } \beta + j \text{ Im } \beta)x} = e^{j(\text{Real } \beta)x} e^{-(\text{Im } \beta)x}$$

with an energy attenuation length given by,

$$L = \frac{1}{2 \text{ Im(}\beta)} \text{ propagation length} \quad (15)$$

Typical values for $L$ are between 20 $\mu$m and 100 $\mu$m.
4.3.D  Effects of different dielectrics on the dispersion curve
4.4 Excitation of SPP at the air-metal interface through prism coupling

We have to address the difficulties in exciting surface plasmons when using free propagating electromagnetic radiation. At a given frequency $\omega$, the magnitude of the $k$ vector (point-A) of a propagating electromagnetic wave (the excitation wave) in air is smaller than the value of the propagating constant $\beta$ of the plasmon (point-B); see Figs. 12 and 13.

The diagram addresses the difficulty of exciting SPP with a propagating field. At a given frequency $\omega$, the wavenumber $\beta_A$ of the free wave propagating in air is smaller than the wavenumber $\beta_B$ of the propagating surface plasmon. Therefore the Snell law (i.e. that the horizontal component of the wavevectors in each medium must be equal) cannot be fulfilled.
Idea: Notice that a free propagating field in silica (point-C in Figure 12) has a wavevector \( \beta_c \) greater than the \( \beta_B \) associated to the plasmon at an air-metal interface. If we consider total internal reflection, the horizontal component of the wavevector \( \beta_C \) could match the value of \( \beta_B \) by choosing a proper incident angle. Then, by total internal reflection, the corresponding evanescent wave (in the air) can then excite the plasmon. This is the strategy introduced by Otto.

4.4.A The Otto configuration
Strategy: Excitation light propagating in a higher index of refraction dielectric (\( n_{\text{glass}} \) for example),
- Total internal reflection established at the higher index/lower-index interface (glass/air interface, for example)
- Evanescent field (in the lower index medium) reaches the lower-index/metal interface to establish a SPP.

Otto showed that one could couple the evanescent wave produced by total internal reflection to the surface mode using the configuration sketched in Fig. 14. Coupling is possible in this case because the wavevector of the light is increased in the denser prism. Provided that the index of refraction of the prism (\( n_{\text{glass}} \)) is higher than the index of refraction of the material in the gap (\( n_{\text{air}} \)), some angle <90° can be found so that \( k_{\text{glass}} \sin \theta = (k_a)_x = \beta \).
What we need METAL glass
AIR glass
Fig. 14 Excitation of SPP at the metal-air interface with an evanescent field. This is accomplished by choosing the proper incident angle in order to match $|k_{\text{glass}}|$ $\text{Sin}\theta$ with $\beta$.

Execution of Otto’s strategy,

Fig. 15 Otto configuration to excite SPP at the air/metal interface.

Since i) the field produced by total internal reflection and ii) the field of the surface plasmon are both evanescent, efficient coupling requires, in addition, that the two surfaces be relatively close. However,
if the surfaces are too close the surface plasmon dispersion is no longer determined by the gap material but also by the prism material.

Hence, maximum coupling occurs at some finite distance from the metal surface and not at contact.

This distinguishes this effect from “frustrated total reflection” phenomena in which the second medium is not a metal but a transparent dielectric. For frustrated total reflection separations of ~50 or 100 Å, effectively physical contact, are needed for extinction. Figure 15 illustrates this difference, showing the reflectivity vs distance curves for a prism above both a silver plate and a glass plate with the same refractive index as the prism.

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**Fig. 2.** Comparison of reflectivity vs gap thickness for attenuated total reflection (ATR), curve 1, and frustrated total reflection (FTR), curve 2, calculated from Fresnel's equations. For ATR the system is glass (BK7) prism, air gap, silver blank; for FTR the system is glass (BK7) prism, air gap, glass (BK7) blank. For both calculations, \( \lambda = 632.8 \text{ nm} \), \( \rho \)-polarized, and \( \phi = 42.9^\circ \).

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**Fig. 16** From Ref: G. T. Sincerbox and J. C. Gordon II, “Small fast large-aperture light modulator using attenuated total reflection,” *Applied Optics* 20, 1491 (1981).
By varying the angle of incidence of the p-polarized radiation at the prism/dielectric interface we vary the x-component of $\mathbf{k}_{\text{glass}}$ and this allows for simple tuning through the resonance.

**Fig. 17** Reflectivity for radiation ($\lambda$=632.8 nm) from thick gold and silver films with a sapphire prism ($n=1.766$). Here the coupling gap $d$ is 0.5 $\mu$m for gold and 1 $\mu$m for silver. From J. R. Sambles, G. W. Bradbery, and F. Yang, "Optical excitation of surface plasmons: an introduction," *Contemporary Physics* **32**, 173 (1991).

**Fig. 18** Variation of surface plasmon resonance (at $\lambda$=632.8 nm) in gold for coupling gaps of (a) 0.5 $\mu$m, (b) 0.75 $\mu$m, (c) 1 $\mu$m. The strength of resonance diminishes rapidly with increasing gap. From J. R. Sambles, G. W. Bradbery, and F. Yang, "Optical excitation of surface plasmons: an introduction," *Contemporary Physics* **32**, 173 (1991).
4.4.B The Kretschmann configuration

The severe distance control required by the Otto geometry led to its limited attention over the years. There is an alternative and much simpler geometry to excite SPP. Rather than using an air gap as spacer (the gap $d$ in Fig. 15 above), Kretschmann and Raether realized that,

the metal itself could be used as the evanescent tunnel barrier, provided it was thin enough to allow radiation to penetrate to the other side.

All that is needed is a prism with a thin (50 nm thick) coating of metal.

![Diagram of Kretschmann and Raether's configuration to excite SPP at the air/metal interface.](image)

**Fig. 19** Kretschmann and Raether’s configuration to excite SPP at the air/metal interface.

A typical result for silver in this geometry is shown in Fig. 20.

![Graph showing surface plasmon resonance for silver film using the Kretschmann and Raether's geometry.](image)

**Fig. 20** Surface plasmon resonance for silver film using the Kretschmann and Raether’s geometry. Note that in this case the critical angle is visible at $\sim 34.5^\circ$. The solid line shows the quality of the fit obtained from the Fresnel theory. From J. R. Sambles, G. W. Bradbery, and F. Yang, "Optical excitation of surface plasmons: an introduction," *Contemporary Physics* **32**, 173 (1991).
Shortcomings of the prism-coupling strategy

SPP excited at the metal-air interface using phase-matching $k_{glass} \sin \theta = \beta$ are inherently leaky waves,

- they lose energy not only due to the inherent absorption inside the metal, but also due to leakage of radiation into the prism.
- the excited plasmon propagation constants lie within the glass light cone.
- The minimum in the intensity of the reflected beam is due to destructive interference between this leakage radiation and the reflected part of the excitation beam.
- For an optimum metal film thickness, the destructive interference can be perfected.

**Fig. 21** Prism coupling and SPP dispersion. Propagation constants between the light lines of air and the prism (usually glass) are accessible; this results in additional SPP damping due to leakage radiation into the prism. The latter is favored by the fact that SPP has a propagation constant $\beta$ that is also a propagating constant for a mode-radiation inside the prism

4.5 Bio-applications of SPP


Concept of surface plasmon resonance biosensing:

Owing to the fact that the vast majority of the field of a surface plasma wave (SPW) is concentrated in the dielectric, the propagation constant of the SPW is...
extremely sensitive to changes in the refractive index of the dielectric. This property of SPW is the underlying physical principle of affinity SPR biosensors.

Biomolecular recognition elements placed on the metal surface recognize and capture analyte molecules present in a liquid sample, which produces a local increase in the refractive index at the metal surface. The increase in refractive index gives rise to an increase in the propagation constant of the SPW propagating along the metal surface (Fig 22), which can be accurately measured by optical means.

![Principle of SPP biosensing.](image)

**Fig. 22** Principle of SPP biosensing.

![Reflectivity and phase for light wave exciting an SPW in the Kretschmann geometry (SF14 glass prism – 50nm thick gold layer – dielectric) versus (a) the angle of incidence for two different refractive indices of the dielectric (wavelength 682 nm), and (b) wavelength for two different refractive indices of the dielectric (angle of incidence 54°).](image)
4.6 SPP and Quantum Entanglement: The quantum nature of SPP


Abstract

The state of a two-particle system is said to be entangled when its quantum-mechanical wavefunction cannot be factorized into two single-particle wavefunctions. This leads to one of the strongest counter-intuitive features of quantum mechanics, namely non-locality[1,2]. Experimental realization of quantum entanglement is relatively easy for photons; a starting photon can spontaneously split into a pair of entangled photons inside a nonlinear crystal. Here we investigate the effects of nanostructured metal optical elements[3] on the properties of entangled photons. To this end, we place optically thick metal films perforated with a periodic array of subwavelength holes in the paths of the two entangled photons. Such arrays convert photons into surface-plasmon waves—optically excited compressive charge density waves—which tunnel through the holes before reradiating as photons at the far side[4–7]. We address the question of whether the entanglement survives such a conversion process.

Our coincidence counting measurements show that it does, so demonstrating that the surface plasmons have a true quantum nature. Focusing one of the photon beams on its array reduces the quality of the entanglement. The propagation of the surface plasmons makes the array effectively act as a ‘which way’ detector.

From a general perspective, the observed conservation of quantum entanglement for the conversion from photon to surface plasmon to photon is a demonstration of the true
quantum nature of SPs. All experiments on SPs so far have not probed their quantum nature, but only their wave nature through their semiclassical dispersion relation. We note that the true quantum nature of the photon was only established in 1977, in antibunching experiments [15,16]. Furthermore, a simple estimate shows that SPs are very macroscopic, in the sense that they involve some 10^{10} electrons. Our experiment proves that this macroscopic nature does not impede the quantum behaviour of SPs, because they can act as intermediates in transmitting entangled photons to yield the expected fourth-order interference.

REFERENCES