Problem #5

**CASE OF NO FRICCTION**

- At B, spring is at its equilibrium position.
- At C, spring is at max compression.
- $Z_0$ is chosen as the zero reference in the Z-axis.

\[ E_A = m g Z_A = (2k_0)(3.8 \text{m/s}^2) \times 2 \text{m} = 39.2 \text{ J} \]

**MOTION**

\[ E_c = K_c + \frac{1}{2} k d_0^2 + m g Z_c \]

If spring at max compression, $K_c = 0$

\[ = \frac{1}{2} \left(\frac{100 \text{N/m}}{m} \right) d_0^2 + m g Z_c \]

\[ = \frac{1}{2} \left(100 \right) d_0^2 + m g Z_c \]

\[ = 100 \times 30^\circ - \frac{1}{2} d_0 \]

\[ = 50 d_0^2 - 9.8 d_0 \]

**MOTION**

If $B \rightarrow C$, mech energy conserves. So $E_c = E_B$

So,

\[ E_B = 39.2 \text{ J} = E_c = 50 d_0^2 - 9.8 d_0 \]

\[ \Rightarrow 50 d_0^2 - 9.8 d_0 - 39.2 = 0 \]

\[ \Rightarrow d_0 = 0.989 \]

Max compression of spring.
CASE OF FRICTION

Work done by the friction force:

\[ W_{\text{non cons}} = \vec{F} \cdot \vec{d} = -f \cdot 4m \]

\[ = -3.39 \text{N m} \cdot 4m \]

\[ W_{\text{non cons}} = -13.56 \text{ joules} \]

\[ E_A = K_A + U_A = 0 + mg z_A = 2 \times 9.8 \times 2 = 39.2 \text{ joules} \]

\[ \text{motion from A to B} \]

\[ E_B - E_A = W_{\text{non cons}} = -13.56 \text{ J} \]

\[ \Rightarrow E_B = E_A - 13.56 \text{ J} = (39.2 - 13.56) \text{ J} \Rightarrow E_B = 25.64 \text{ J} \]

Knowing the energy at joint B, we can calculate now how much will the spring compress.
motion
From B $\rightarrow$ D:
the work done by the
friction force (no consen.) is
\[ W_{\text{fric}} = f \cdot d_i \]
\[ = f \cdot d_i \cdot \cos \theta \]
as before \( f = \mu N \)
\[ = \mu k \cdot mg \cdot \cos 30^\circ \]
\[ W_{\text{fric}} = -3.39 \text{ Nm} \cdot d_i \]
d_i is unknown yet.

\[ E_0 = K_0 + \frac{1}{2} k d_i^2 + mg d_i \]
\[ \text{at max} \quad \text{compression} \quad = -d_i \cdot \sin 30^\circ \]
\[ = -\frac{1}{2} d_i \]

\[ E_0 = \frac{1}{2} k d_i^2 - \frac{1}{2} mg d_i \]

And we already know \( E_B = 25.64 \text{ J} \)

\[ E_0 - E_B = \frac{1}{2} k d_i^2 - \frac{1}{2} mg d_i - 25.64 \text{ J} \]

Using \[ E_0 - E_B = W_{\text{non consen.}} \]
we obtain:
\[ \frac{1}{2} k d_i^2 - \frac{1}{2} mg d_i - 25.64 = -3.39 d_i \]

\[ 50d_i^2 - 9.8d_i - 25.64 = -3.39d_i \]

\[ 50d_i^2 - 6.41d_i - 25.64 = 0 \]
\[ \Rightarrow d_i = 0.783 \text{ m} \]
\[ W_{\text{NON}} = -3.39 \text{ Newton} \times d_1 \]
\[ = -3.39 \text{ Newton} \times 0.783 \text{ m} \]
\[ W_{\text{NON}} = -2.65 \text{ Joules} \]

From \( B \rightarrow D \)

Motion
\( B \rightarrow D \rightarrow B \)

At \( B \), block starts with \( E_B = 25.654 \text{ J} \)

In the round trip \( B \rightarrow D \rightarrow B \), the work done by the friction is \( W_{\text{NON}} = -2 \times (-2.654 \text{ Joules}) = 5.31 \text{ Joules} \)

therefore \( E_B = 25.654 \text{ J} - 5.31 = 20.33 \text{ Joules} \)

\[ E_B = 20.33 \text{ J} \]

Motion
\( B \rightarrow G \)

\[ x = ? \]
\[ E_g - E_{f3} = \mathcal{W}_{non} \]
\[ = -3.39 \, J \]

\[ E_g = K_0 + mgz \overset{v = 0}{\rightleftharpoons} \frac{1}{2}mgx = 9.8 \, J \]
\[ x = \frac{v}{2} \sin 30^\circ \]
\[ m = 2 \, kg \]

\[ E_g = 20.33 \, J \]

\[ \Rightarrow E_g - E_b = 9.8 \, x - 20.33 \, J \]

\[ E_g - E_b = \mathcal{W}_{non} = -3.39 \, J \]

\[ \Rightarrow 9.8 \, x - 20.33 \, J = -3.39 \, J \]
\[ 13.2 \, x = 20.33 \Rightarrow x = 1.54 \, m \]
Problem 6

Part 3

Problem involves only internal forces

\[ X_{cm} \text{ with respect to } "0" \]

\[ X_{cm} = \frac{80 \times 0 + 60 \times 0 + 120 \times 2}{260} \]

\[ = \frac{12}{13} \text{ meters} \hspace{1cm} (1) \]

\[ m_1 \text{ and } m_2 \text{ interchange seats} \]

\[ X'_{cm} = \frac{80 (5+2) + 60 (5) + 120 (3)}{260} \]

\[ = \frac{160 + 260 \times 5}{260} \]

\[ = \frac{8}{13} + 5 \hspace{1cm} (2) \]

Since there are not external forces acting on the system \((m_1, m_2, \text{boat})\), \(\vec{V}_{cm}\) remains constant.

\(\vec{V}_{cm}\) was zero before, it will be zero after the seat interchange.

\(\vec{V}_{cm} = 0\) implies that \(X_{cm}\) remains constant.
therefore
the values of $x_{cm}$ and $x'_{cm}$, given in
expression (1) and (2), must be equal

\[
\frac{12}{13} = \frac{8}{13} + \delta
\]

\[
\Rightarrow \delta = \frac{4}{13} = 0.31 \text{ m}
\]

\textbf{Part 2}

\[X_{cm} = \frac{120 (1) + 80 (-1) + 60 (8)}{260}\]

\[= \frac{4d}{260} = \frac{2}{13} \text{ m} \quad (1)\]

\[X_{cm} = \frac{120 (-18) + 80 (18) + 60 (8)}{260}\]

\[= -\frac{120 + 80 + 260 \delta}{260}\]

\[= -\frac{4d}{260} + \delta = -\frac{2}{13} + \delta \quad (2)\]

(1) = (2) implies

\[
\frac{2}{13} = -\frac{2}{13} + \delta \Rightarrow \delta = \frac{4}{13} \text{ m}
\]
If the boy is still on the surface,

then

\[ \frac{mv^2}{R} = \text{centrifugal force} = mg \cos \theta - N \]

\[ R \cos \theta = h \]

\[ \frac{mv^2}{R} = mg \frac{R \cos \theta}{R} - N \]

\[ \left( \frac{mv^2}{R} = mg \frac{h_0}{R} - N \right) \]

\[ E_n = \frac{1}{2} mv^2 + mg R \quad \text{small hill} \]

\[ E_0 = \text{mg} R \]

\[ \left\{ \begin{align*}
  & \text{conservation of mechanical energy} \rightarrow \\
  & mgR = \frac{1}{2} mv^2 + mg h
\end{align*} \right. \]
\[ m v^2 = 2 g y (r-h) \Rightarrow v^2 = 2 (r-h) g \]

\[ \frac{v^2}{R} = 2 \frac{r-h}{R} g \]

\[ \frac{m v^2}{R} = m 2 \frac{r-h}{R} g \] \hspace{1cm} (2)

\[ 2 m \frac{r-h}{R} g = m g \frac{h}{R} - N \]

\[ \text{motion} \]

Condition for the boy to leave the ice-circumference:

\[ N = 0 \]

\[ \Rightarrow 2 \frac{(r-h) g}{R} = \frac{g h}{R} \Rightarrow 2 (r-h) = h \]

\[ 2r - 2h = h \]

\[ 2r = 3h \]

\[ \frac{2}{3} R = h \]