CHAPTER 5: Force and
The Newton's Laws of Motion

DYNAMICS

- What causes an acceleration?
  - Galileo (1564-1643) observations
    Motion of a particle with no friction
  - Newton
    Newton's First Law
    - What does it mean zero total force
    - What is an inertial system of reference
  - Concept of mass
  - Concept of force
- Newton's Second Law \( F = ma \)
- Newton's Third Law \( F = -F' \)
The horizontal motion is independent of the vertical motion.

Whether the block follows paths I, II, or III depends on the magnitude of $v_0$.

Path I: acceleration = ?
H: $a =$ ?
III: $a =$ ?

The horizontal motion is independent of the vertical motion.

\[ x = x_0 + v_{x0} t \]
\[ y = y_0 + v_{y0} t - \frac{1}{2} g t^2 \]
Review: Uniform circular motion

- Magnitude of the velocity $|\vec{v}| = V$ is constant throughout the motion.

- Acceleration always points toward the center of the circumference

$$a = \frac{V^2}{R}$$
Velocity is not constant

If the block experience an acceleration, "something" must be responsible for it.
We invent the concept of "force."

Intuitively, we associate the action of forces whenever the velocity of the particle changes.
Galileo (1564-1643) extrapolated that in a frictionless table, the particle would continue moving at constant velocity.

Prior to Galileo, it was thought wrongly that objects naturally tend to reach a final zero velocity if no force were acting on them. To keep objects moving at constant velocity one actually needs to apply a force.
Newton's First Law: (version-3)

An object at rest stays at rest unless acted on by an external force.

An object in motion continues to travel with constant velocity unless acted on by an external force.

But, didn't we say that velocity is a relative quantity?
In the statement (1), what reference is being used?

If you were at a reference, from which you observe that a particle moves with constant velocity when no force acts on it, then you are at an inertial system of reference.

That is, Newton's First law serves to define an inertial system of reference. You may frequently hear "those references in which Newton's First law is found to be valid are called inertial systems of reference."
Not all reference frames are inertial reference frames

Observations made using a reference attached to the car will not follow Newton's First Law

Observations made from a reference attached to a spinning turn table will not follow Newton's First Law
In an inertial system of reference: if no force at all acted on a particle, then such particle will continue its motion under constant velocity.

NEWTON's FIRST LAW

In reality, there exist approximate inertial systems of reference; some better than other ones.

- Earth
- Sun
- Galaxy
The concept of FORCE

- The elongation of the spring allows us to compare force strengths.

Bigger force needed to produce the same acceleration on bigger masses.

- Forces in nature
  - Gravitational
    \[ F_g = \frac{G M m}{r^2} \]
  - Electro-magnetic
  - Strong forces (nucleus)
    \[ F_{strong} \]
  - Electro-weak forces
The concept of MASS

- Scalar (i.e. #) that indicates how sluggishly the particle moves in response to forces

- The concept of mass was invented to describe how a particle responds to the action of forces

\[ m, \quad M_2 \quad m < M_2 \]
NEWTON's SECOND LAW

- A force causes the acceleration of a particle

- Observation:
  A force \( F_2 \) of double strength causes double acceleration on the same standard mass

\[
\frac{F_2}{F_1} = 2 \quad \Rightarrow \quad \frac{a_2}{a_1} = 2
\]

In general

\[
\frac{F_2}{F_1} = \frac{a_2}{a_1} \quad \text{(for const mass)}
\]

- Another observation

\[
\frac{m_2}{m_1} = 2 \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{2}
\]
In general \[ \frac{m_1}{m_2} = \frac{a_2}{a_1} \] (for constant force)

**Mass**: An intrinsic characteristic of a body
- It comes with its existence

Mass \( \neq \) weight
\( \neq \) density

Mass: Abstract concept defined by experiment

“We do not know what gives rise to this property of inertia. Whatever this inherent inertial characteristic of matter really is, we call it mass”
All these experimental observations lead to:

\[ \sum F = ma \]

Total external force acting on the mass \( m \)

Unit of force:

\[ \text{kg \cdot m/s}^2 = \text{N} \]

1 Newton

**FINAL REMARKS:**

The following concepts are interrelated:

- **FORCE**
- **INERTIAL SYSTEM OF REFERENCE**
- **MASS**

The definition of one of them, without invoking the other one, is a fruitless effort. We have to learn about their meaning simultaneously.
Optional reading

Einstein's modification to Newton's law

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

where

- \( m_0 \): is the "rest mass"
- It represents the mass of a body that is not moving
- \( c \): Speed of light \( 300,000 \text{ km/s} \)

Notice:

At low speeds \( v = 100 \text{ km/h} \)
the factor \( \left( \frac{v}{c} \right)^2 \) is very small, \( \sim 10^{-14} \),
which gives

\[ \frac{\Delta m}{m_0} \sim \frac{1}{2} \times 10^{14} \]

Exceedingly small change to be perceived by the naked eye.
You may wonder what does the speed of light have to do in this description of mechanical motion.

It turns out, all comes from Einstein’s work in the Special Theory of Relativity, which has the following postulates:

(1) The fundamental laws of physics must be the same in all INERTIAL SYSTEM OF REFERENCE.

(2) The speed of light in vacuum has the same numerical value c when measured in any INERTIAL SYSTEM OF REFERENCE, independent of the source and observer.

(Note: Two inertial system of reference move with respect to each other at constant velocity.)
The first postulate is somewhat related, although it is not exactly the same, with Newton's First Law. Two observers that move relative to each other at constant velocity should see the same physics.

It turned out that, initially, the laws of electricity and magnetism given by the Maxwell Equations (ME) appeared to contradict the first postulate (which, let's remember, was conceived through the description of mechanical motion).

This situation triggered efforts to modify Maxwell Eq., but predictions turned out wrong.

Einstein suggested that the Maxwell Eq. were right, and what had to be modified were the mechanical laws of motion.

\[ m_0 \rightarrow \frac{m_0}{\gamma} \]
He postulated that the speed of light was the same in any inertial system of reference (which defies common sense if you think about it).

\[ x', y', z', t' \]

Invariance of the speed of light required that the coordinates \((x, y, z, t)\) be related to \((x', y', z', t')\) in a peculiar way.

For example,

\[ x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]
When this transformation of coordinates were applied, by Einstein, to describe mechanical motion, the result was that everything "looked" the same, provided that

\[ m_0 \rightarrow \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]

Ref: Feynman Lectures, Vol. I
Sections 15-1, 15-2, 15-8
- R. Resse, University Physics
  Section 25-3
Example
Two forces act on the block shown in the figure

3 Kg

Force $\vec{F}_1$: $F_1 = 9.0 \text{ N}$
acting due east

Force $\vec{F}_2$: $F_2 = 8.0 \text{ N}$
acting 62° north of west

Find the acceleration of the block

Solution

Let's find the total vector force acting on the block

$\vec{F}_1 = 9 \text{ N} \hat{i}$

$\vec{F}_2 = -8 \text{ N} \cos(62°) \hat{i} + 8 \text{ N} \cos(62°) \hat{j}$

Total force $\sum \vec{F} = \vec{F}_1 + \vec{F}_2$

$= [9 - 8 \cos(62°)] \text{ N} \hat{i} + 8 \text{ N} \sin(62°) \hat{j}$

Newton's second law $\sum \vec{F} = m \ddot{\vec{x}}$ implies

$[9 - 8 \cos(62°)] \text{ N} \hat{i} + 8 \text{ N} \sin(62°) \hat{j} = m \left( \alpha_x + \alpha_y \right)$
\[ a_x = \frac{9 - 8 \cos 62^\circ}{3} \frac{N}{kg} \]
\[ a_y = \frac{8 \sin 62^\circ}{3} \frac{N}{kg} \]

\[ \overrightarrow{F_{\text{Total}}} = \sum \overrightarrow{F} = 5.2 \ N \hat{x} + 7.1 \ N \hat{y} \]
\[ \overrightarrow{a} = 1.7 \ \frac{m\hat{x}}{s^2} + 2.3 \ \frac{m\hat{y}}{s^2} \]

Symbol to indicate that we are adding all the forces.

**Question:** Could we have a situation like this?
You may wonder, what is going on in the following case:

The block moves horizontally, so the acceleration must obviously point in the horizontal direction.

\[ \vec{a} \]

How come?

**Answer**

\[ \vec{F} \] is not the total force acting on the block

- \[ \vec{F} \] (reaction from the floor)
- \[ \vec{N} \] (weight)
- \[ \vec{w} \] (Earth's gravitational force)

(Earth attracts the block down)
\[ \vec{F}_{total} = \vec{F} + \vec{N} + \vec{W} \]

Thus, \( \vec{F}_{total} \) is parallel to \( \vec{z} \), as should always be!
Particular Case: $\sum F = 0$  
Equilibrium

If $\sum F = 0$
then $\ddot{a} = 0$
then $\vec{v} = \text{const}$

$\Rightarrow$ $\vec{v}$ could be $0$
$\Rightarrow$ $\vec{v}$ could be $\neq 0$

This is: If the total force acting on a particle is ZERO, it doesn't mean that the particle is at rest necessarily.

Example:

\[ F_1 = 3\text{N} \quad F_2 = 5\text{N} \quad F_3 \quad \]

What should be $F_3$ such that

a) Block remains stationary

b) Block moves to the right with constant speed of 5 m/s
\[ \sum F = 0 \quad \Leftrightarrow \quad \text{equilibrium} \]

("equivalent")

Example

Find the tension on the string.

Forces acting on "M"?

In this case \( \sum F \) means \( (\vec{W} + \vec{T_2} + \vec{T_3}) \)
Along \( X \): \( (T_2 \cos 60^\circ) + (-T_1 \cos 30^\circ) = 0 \)

Along \( Y \): \( (T_2 \sin 60^\circ) + (T_1 \sin 30^\circ) + (-W) = 0 \)

solution

\[
\begin{cases}
\frac{1}{2} T_2 - \frac{\sqrt{3}}{2} T_1 = 0 \\
\frac{\sqrt{3}}{2} T_2 + \frac{1}{2} T_1 - W = 0
\end{cases}
\]

\[
\frac{\sqrt{3}}{2} (\sqrt{3} T_1) + \frac{1}{2} T_1 = W \Rightarrow 2 T_1 = W \\
T_1 = \frac{W}{2}
\]

\[
T_2 = \frac{\sqrt{3}}{2} W
\]
NEWTON's THIRD LAW

- Forces are always in pairs.
- We said before that, in order to disturb the uniform linear motion of a particle, a force needs to be applied.

- We can perturb particle A's motion by placing, for example, another magnet B nearby.
So far, we have been saying: Magnet B exert a force $\overrightarrow{F_{AB}}$ on the smaller magnet A.

Newton's third law states that: Simultaneously, with the action of force $\overrightarrow{F_{AB}}$, there exist another force $\overrightarrow{F_{BA}}$ acting on magnet B.
**NEWTON's THIRD LAW**

**Notice:**

\[ \vec{F}_{AB} \text{ acts on particle } A \]

\[ \vec{F}_{BA} \text{ acts on particle } B \]

**This is,**

action force \( \vec{F}_{AB} \) and
reaction force \( \vec{F}_{BA} \) act on different bodies

**Also,**

\[ \vec{F}_{AB} = - \vec{F}_{BA} \]

**NEWTON's THIRD LAW**
Hint: Whenever we say that a given force ($F_{\text{action}}$) is acting on a given particle A, be ready to identify the reaction force ($F_{\text{reaction}}$) that particle A exerts back to the source that is disturbing its motion.

Example: An orbiting satellite

Earth exerts a force on the satellite

Force that the satellite exerts on the Earth
Example

\[ \vec{F}_1 \text{ attractive force the sun exerts on the Earth} \]
\[ \vec{F}_2 \text{ attractive force the Earth exerts on the sun} \]

In other words:
The SUN affects the motion of the earth
\[ \vec{F}_1 = m_{\text{earth}} \vec{a}_{\text{earth}} \]
The EARTH affects the motion of the SUN
\[ \vec{F}_2 = M_{\text{sun}} \vec{a}_{\text{sun}} \]
Comparison: (Rough estimation)

Since $|F_1| = |F_2|$ we have

$$\frac{M_{\text{Earth}} \cdot a_{\text{Earth}}}{\text{Small}} = \frac{M_{\text{Sun}} \cdot a_{\text{Sun}}}{\text{Big}}$$

Actually

$$\frac{6 \times 10^{-24}}{2 \times 10^{30}} = \frac{M_{\text{Earth}}}{M_{\text{Sun}}} = \frac{a_{\text{Sun}}}{a_{\text{Earth}}}$$
EXAMPLE

We'll learn how to deal with tension forces along ropes.

* Start from the bottom part.
Let's establish all the action and reaction forces first (see arrows)

**Roof**

![Diagram of forces](image)

- **T2'** Force due to the ceiling acting on rope-2
- **T2** Force on B due to rope-2
- **T1 = T1'** Force on B due to rope-1
- **T1 = T1'** Force on rope-1 due to B
- **T1** Force on rope-1 due to A
- **T1** is the force on A due to rope-1

**Equilibrium of block-B implies:**

\[ T_2 = m_B g + T_1' \]

**Equilibrium of block A implies:**

\[ T_1 = m_A g \]
Thus,

\[ T_2 = m_b g + T_1 \]
\[ = m_0 g + r_i = m_b g + m_a g \]

\[ T_1 = m_a g \]
Example

"ATWOOD'S MACHINE"

Approximation: No friction force

Approximation: Cord is massless
(Or its mass is very small compared to M and m)

Approximation: No friction force

Since the cord is massless:

\[ T = T' \]

If the cord does not elongate, then:

\[ |\vec{a}_m| = |\vec{a}_M| \]

Let's call it \( a \)
\[ T - mg = ma \]

Adding up these two expressions, we obtain

\[ Mg - mg = (m + M)a \]

\[ \Rightarrow a = \frac{M - m}{M + m} g \]