**ELECTRIC FIELD**

If an electric charge \( q_o \) located at a position \( P \) experiences a force, we say that there exist an electric field in a region around \( P \).

The electric field is produced by the system of charges. The electric field characterizes the system of charges.

Operational procedure to calculate the ELECTRIC FIELD produced a given system of charges at the point \( P \)

1. Place a test charge \( q_o \) at the point \( P \).

2. Find the electrical force \( F \) that such system of charges exerts on \( q_o \).

3. The ELECTRIC FIELD at \( P \) will be given by

\[
\frac{\vec{F}}{q_o} = \vec{E}
\]

The ELECTRIC FIELD is a vector
First, some basic concepts about vectors

Given the vector \( \vec{A} \)

\[ \vec{B} = -\vec{A} \]

\[ \vec{C} = 4\vec{A} \]

\[ \vec{D} = \frac{\vec{A}}{-4} \]

\[ \vec{B} = -\frac{1}{4}\vec{A} \]

Next, we show that the electric field produced by a positive charge \( Q \) is independent of the sign of the test charge used to evaluate it.
Now we show that the electric field produced by a negative charge $q$ is independent of the sign of the test charge used to evaluate it.

Positive test charge $q_0$

$\vec{F} = \frac{\vec{F}}{q_0}$

Negative test charge $q_1$

$\vec{F} = \frac{\vec{F}}{q_1}$

Positive test charge $q_3$

$\vec{F} = \frac{\vec{F}}{q_3}$

Negative test charge $q_4$

$\vec{F} = \frac{\vec{F}}{q_4}$
First, let's place a positive test charge \( q_0 \) at the position indicated by \( P \), and evaluate the total electric force acting on that charge.

Example. Indicate the electric field at the point \( P \) established by the two positive charges shown in the figure. Notice, there is no charge at \( P \).

Next, we simply calculate the ratio \( \frac{\vec{F}}{\vec{q}_0} \). (Since \( q_0 \) is positive, then \( E \) and \( F \) will be oriented in the same direction.) Notice, there is no charge at \( P \).
Had we have chosen a negative test charge $q_o$, we would have obtained the following

\[ \mathbf{E} = \frac{\mathbf{F}}{q_o} \quad \text{(Since $q_o$ is negative, then $\mathbf{E}$ and $\mathbf{F}$ will be oriented in the opposite direction)} \]

**Conclusion**

$\mathbf{E}$ does not depend on the "test" charge $q_o$. 

System of charges
**Exercise.** Find the electric field at point $P(0,4)$ and at the point $T(0,-2)$

$$q_1 = 10 \mu C$$
$$q_2 = -10 \mu C$$

**Solution**

$$E_{01} = 9 \times 10^9 \frac{(10 \times 10^{-6})}{(5 \times 10^{-2})^2} = 3.6 \times 10^7 \text{ N/C}$$

Unit vector $u_1 = (3/5, 4/5) = (\cos 53^0, \sin 53^0)$

$$E_{02} = 3.6 \times 10^7 \text{ N/C}$$

Unit vector $u_2 = (3/5, -4/5)$

$$E \text{ (at } P) = 2 \times 3.6 \times 10^7 \text{ N/C} \times (3/5, -0) = (4.32 \times 10^7 \text{ N/C, 0})$$
Does the electric field exist at only one point?

No.
The electric field exists at every point of the space surrounding the system of charges.

Does the electric field depend on the magnitude or sign of the test charge $q_0$?

No.
The electric field exists independent of the test charge $q_0$.
We have used $q_0$ only as a convenient way to illustrate how can we measure the electric field (experimentally).
How can we mathematically verify that the electric field of a system of charges is independent of the test charge $q_0$ alluded in the experimental procedure given at the beginning of this chapter?

To "see" this, let's use Coulomb's law:

$$\vec{F} = \sum_i \frac{1}{4\pi \varepsilon_0} \frac{q_i q_0}{r_i^2} \hat{U}_i$$

Notice that each term in the sum has the factor $q_0$.

Since $q_0$ is constant, we can factor it out:

$$\vec{F} = q_0 \sum_i \frac{1}{4\pi \varepsilon_0} \frac{q_i}{r_i^2} \hat{U}_i$$

Resulting

$$\vec{E} = \frac{\vec{F}}{q_0} = \sum_i \frac{1}{4\pi \varepsilon_0} \frac{q_i}{r_i^2} \hat{U}_i$$

$\vec{E}$ is independent of $q_0$. 

System of charges
(Charges on a stick bar)
Summary

If a charge $q$ (positive or negative) were placed at the position $P$ shown in the figure above (and provided that such charge does not disturb the position of the system of charges) then $q$ will experience a force given by,

$$F = q \hat{E}$$

System of charges

$$\vec{E} = \sum_{i} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \hat{u}_i$$

which is written in a more compact form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^2} \hat{u}_1 + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2^2} \hat{u}_2 + \ldots$$
Here we observe an object of charge $q$. If there is NOTHING ELSE in the surrounding environment, HOWEVER, we say that THERE EXISTS an electric field SURROUNDING THE CHARGE $q$. 
How to calculate the electric field $E$ at a point $P$ located a distance "$r$" from a positive point-charge of magnitude $e$ ?

$$\vec{E} \text{ (at } P) = \frac{1}{4\pi \varepsilon_0} \frac{(e) \hat{\mu}}{r^2}$$

Units

- $e$: coulomb
- $r$: meters
- $E$: Newtons/coulomb.

How to calculate the electric field $E$ at a point $S$ located a distance "$r$" from a negative point-charge of magnitude $e$ ?

$$\vec{E} \text{ (at } S) = \frac{1}{4\pi \varepsilon_0} \frac{(-e) \hat{\mu}}{r^2}$$
Example: Electric field produced by a positive point-charge $q$

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{q_0 q}{r^2} \]

$F$ in Newtons

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \]

$E$ in \( \text{Newton} \)s \( / \) \( \text{Coulomb} \)

Notice $E$ depends on $r$

$r =$ radius
Notice, the Electric Field at positions farther away from $q$ are weaker.

$r_1$ and $r_2$ are radius.

Usually, the figure above is rather shown as the diagram below.

Both figures 1 and 2 are alright.
Example: Electric field produced by a negative point-charge $Q$
What is a DIPOLE?

2 charges of equal magnitude but different sign, and separated a distance $d$

$-q \quad \vec{d} \quad +q$

Definition of the electric dipole moment:

$\vec{p} = q \vec{d}$

$\vec{p}$ is a vector that point from the negative charge towards the positive charge.

Microscopic description of non-conductivity, in terms of the electric-dipole concept

Charge induction inside the molecule results in the creation of a dipole.

The dipole orients in response to the external electric field.
The dielectric breakdown

There exists a maximum value for the external electric field beyond which the atoms inside the material become ionized.

The dislodged electrons (now free) give rise to a high current, which heats up, and eventually destroy, the material. This phenomenon is called dielectric breakdown.

Dielectric breakdown occurs in air at $E = 3 \times 10^6 \text{ N/C}$.
Accordingly, the molecule of water can be treated as an electric dipole

\[ p = 6.2 \times 10^{-30} \text{ Cm} \]

For a neutral water molecule in its vapor state.
Real structure

10 protons (+)
10 electrons (-)

Simplified model

\[ p = Q \cdot d = 6.2 \times 10^{-30} \text{ Cm} \]
\[ = (10e) \cdot d \]

\[ d = \frac{6.2 \times 10^{-30} \text{ C} \cdot \text{m}}{10 \times 1.6 \times 10^{-19} \text{ C}} \]
\[ \approx 4 \times 10^{-12} \text{ m} \]

d = 0.04 Angstroms
Electric field established by a dipole

\[ \vec{E} = \frac{\vec{E}}{\varepsilon_0} \]

Electric field at different locations (P, T, S) produced by the charges +Q and -Q

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_1^2} \hat{u}_1 + \frac{1}{4\pi\varepsilon_0} \frac{-Q^2}{r_2^2} \hat{u}_2 \]
Notice: Electric field lines leave from positive charges and ends on a negative charge.

Question: Is the magnitude of the electric field constant along a field line?
Charges inside an electric field

Given the following electric field

what will happen to a charge \( q_0 \) placed inside the electric field?

If the local electric field (at the position where the charge \( q_0 \) is located) has a value \( \vec{E} \), then the charge \( q_0 \) will experience a force

\[
\vec{F} = q_0 \vec{E}
\]

Question:
Is the electric field shown in the figures above uniform?
Charge moving inside a uniform electric field

\[ E = 1.4 \times 10^6 \text{ N/C} \]

Negative charge of mass \( m \)

At \( t = 0 \), \( \vec{V} = \vec{V}_0 \)

Question: What would be trajectory followed by the charged particle?

Does the acceleration of the particle change during its the motion inside the field?

Question: If \( m = 1.3 \times 10^{-10} \text{ kg} \), \( |q| = 1.5 \times 10^{-13} \text{ C} \), and \( \vec{V}_0 = 18 \text{ m/s} \)

Calculate:

i) The acceleration of the particle

ii) How much does the particle deflects (vertically) when it travels a horizontal distance of 1.6 cm from the instant it enters the region of the electric field?
Electric-dipole inside an electric field

\[ \vec{F} = q \vec{E} \]

\[ \vec{F}_r = (-q) \vec{E} \]
Order of magnitude of the electric force

Repulsive force between 2 protons inside a nucleus

\[ F = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19} \text{C})^2}{(4 \times 10^{-15} \text{m})^2} \]

\[ = 14 \text{ Newtons} \]

Attractive force between a proton in the nucleus of an atom and an electron flying around the nucleus

\[ F = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19} \text{C})^2}{(10^{-10} \text{m})^2} \]

\[ = 23 \times 10^{-9} \text{ Newtons} \]

Electric field established by a proton at a distance 1 Angstrom far away

Charge of a proton: \(5.6 \times 10^{-19} \text{C}\)

\[ E = 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{(1 \times 10^{-10} \text{m})^2} \]

\[ = 19 \times 10^6 \frac{\text{Newton}}{\text{Coulomb}} \]
**Electrical Ground:** A very large conductor able to supply an unlimited amount of charge
Exploiting symmetry (of the charge distribution in a system of charges) to calculate electric fields

Example 1. Electric field established by a uniformly charged circular line.

A total amount of positive charge $Q$ is UNIFORMLY distributed along a plastic ring of radius $R$.

Question: What is the direction of the electric field at point $P$?
Solution

First: Define a linear charge density $\lambda$

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi R}$$

Second: Divide the plastic ring into small segments of length $ds$

The amount of charge $dq$ contained in each segment of length $ds$ is

$$dq = \lambda \, ds$$

Third: Apply symmetry

For each small segment containing a charge $dq_1$, there exists another segment symmetrically located containing a charge $dq_2$ (with $dq_2 = dq_1$) such that the HORIZONTAL components of the electric field cancel out. That is, only the VERTICAL components make a net contribution to the TOTAL FIELD.
Thus, based on the grounds of symmetry, we calculate the VERTICAL component of the electric field produced by a charge \( dq \) contained in a segment of length \( ds \), and then ADD UP all those vertical components correspondent to each charge on the circular line.

\[
dE = \frac{1}{4\pi \epsilon_0} \frac{dq}{R^2 + z^2}
\]

but \( dq = \lambda \, ds \)

\[
dE = \frac{1}{4\pi \epsilon_0} \frac{\lambda \, ds}{R^2 + z^2}
\]

\( (dE) \cos \theta \) \text{ vertical component of the electric field}

where \( \cos \theta = \frac{z}{\sqrt{R^2 + z^2}} \)
\[ E = \int_{d\varepsilon \cos \theta}^{\text{vertical component of the electric field}} \frac{z \lambda}{4\pi \varepsilon_0 (R^2 + z^2)^{3/2}} ds \]

Adding up the contribution from all the segments \( ds \) we obtain

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{2\lambda}{(R^2 + z^2)^{3/2}} \int ds \]

\( \text{integral of all over the ring} \)

= \( 2\pi R \)

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{\lambda 2\pi R z}{(R^2 + z^2)^{3/2}} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{z Q}{(R^2 + z^2)^{3/2}} \]
Checking our results with physical arguments

Physics: What should be the value of the electric field at $z = 0$?

Math: Does our math result agree with our physics argument?

$$E = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot Q}{(R^2 + z^2)^{3/2}} \quad \text{when} \quad z \to 0$$

Physics: What to expect when $z \to \infty$?

How should the electric field vary for large values of $z$?

Math: Does our math result agree with physics argument? Let's take $z \to \infty$

$$E = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot Q}{(R^2 + z^2)^{3/2}} \quad \text{when} \quad z \to \infty$$

$$\to \frac{1}{4\pi \varepsilon_0} \frac{z \cdot Q}{(z^2)^{3/2}} = \frac{1}{4\pi \varepsilon_0} \frac{z \cdot Q}{z^3}$$

$$= \frac{1}{4\pi \varepsilon_0} \frac{Q}{z^2}$$
Example 2. Electric field established by a uniformly charged disk.

A total amount of charge $Q$ is uniformly distributed on its surface.

Let's define a surface charge density $\sigma$

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

Area of the disk $A = \pi R^2$

Question: What is the direction of the electric field at point P?
Strategy: We will exploit the fact that we already know the electric field produced by a charged ring.

We will consider the disk as a collection of rings (each having different radius).

Ring of radius $r$ and thickness $dr$

How much charge ($dq$) does this ring contain?

Area of the Ring:

$$dA = 2\pi R \, dr$$
\[ dq = \int \frac{\partial f}{\partial \text{area}} = \int 2\pi r \, dr \]

surface charge density

We know the electric field \( dE \) produced by a ring of radius \( r \) containing an amount of charge \( dq \)

\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{2}{(r^2 + z^2)^{3/2}} \frac{\partial f}{\partial \text{area}} \]

magnitude of the electric field due to a ring of radius \( r \) and thickness \( dr \)

\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{2 \, r \pi r \, dr}{(r^2 + z^2)^{3/2}} \]

Total electric field is the addition of the electric fields produced by all the rings of different radius, from \( r = 0 \) to \( r = R \)
Total \[ E = \int \frac{1}{4\pi \varepsilon_0} \frac{2 \sigma 2\pi r \, dr}{(r^2 + z^2)^{3/2}} \]

What is (are) the variable(s) in the integral?

\[ \sigma, r, z \]

\[ E = \frac{2 \sigma}{4 \varepsilon_0} \left( \int_{r=0}^{r=R} \frac{2 \pi \, dr}{(r^2 + z^2)^{3/2}} \right) \]

\[ = \frac{2 \sigma}{4 \varepsilon_0} \left[ -2 \frac{3}{(r^2 + z^2)^{1/2}} \right]_{r=0}^{r=R} \]

\[ = \frac{\sigma}{2 \varepsilon_0} \left( -2 \frac{3}{(R^2 + z^2)^{1/2}} - \frac{-2}{z} \right) \]

\[ E = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{z}{(R^2 + z^2)^{1/2}} \right) \]
"Checking our results.

Physics: What to expect when $z \to \infty$?
What should be the dependence of $E$ with respect to the variable $z$ for large values of $z$?

Math: $E = \frac{\sigma}{2 \varepsilon_0} \left(1 - \frac{z}{(R^2 + z^2)^{1/2}}\right)$

\[\text{when } z \to \infty\]

\[\frac{\sigma}{4 \varepsilon_0} \frac{R^2}{z^2}\]

homework
See sample problem 23-6

\[\text{since } \sigma = \frac{Q}{\pi R^2}\]
we obtain

$E \to \frac{1}{4\pi \varepsilon_0} \frac{Q}{z^2}$

* Hint: For $E \ll 1$ ($E$ much smaller than $Q$)

$(1 + \varepsilon)^p \approx 1 + p\varepsilon$

$p$ can be any real number (positive, negative, fraction, etc.)
Checking our results

What is the value of the electric field when $R \to \infty$?

This is, when the disk becomes an infinite charged plate.

\[ E = \frac{\varphi}{2E_0} \left( 1 - \frac{z}{(R^2 + z^2)} \right) \quad R \to \infty \quad \frac{\varphi}{2E_0} \]

\[ E = \frac{\varphi}{2E_0} \quad \text{for a uniformly charged infinite sheet} \]