Example: Oscillating plank
The figure shows a horizontal planks of length L=50 cm, and mass M= 1 Kg, pivoted at one end. The planks’ is also supported by a spring at 2/3 of its length, as shown in the figure; the spring constant has a value of k =10 N/cm.

a) When in equilibrium, the plank is horizontal. Evaluate the length $\delta$ the spring is compressed in this equilibrium state.

b) Assuming that the plank undergoes small amplitude oscillations, calculate the period and frequency of those oscillations.
When in equilibrium, the frame is horizontal. In this situation, the spring is compressed a length $s$.

\[ Mg \frac{1}{2} = ks \frac{2}{3} L \]

\[ \Rightarrow \frac{3}{4} \frac{Mg}{k} = s \]

\[ k = \frac{10 N}{cm} = \frac{10 N}{10^{-2} m} = 1000 N/m \]

\[ I_{cm} = \frac{1}{12} ML^2 \]

\[ I_{pivot} = \frac{L}{12} ML^2 + Mh^2 \]

\[ h = \frac{l}{2} \]

\[ = \frac{L}{12} ML^2 + M(\frac{l}{2})^2 \]

\[ = \frac{L}{12} ML^2 + \frac{1}{4} ML^2 \]

\[ I_{pivot} = \frac{3}{4} ML^2 \]
Torque

\[ \text{Torque due to gravity} = \frac{Mg}{2} \cos \theta \]
\[ \approx \frac{Mg}{2} \frac{L}{2} \]
for small \( \theta \)

\[ \text{Torque due to string} = k(y - \delta) \frac{2}{3} L \cos \theta \]
\[ \approx -k(y - \delta) \frac{2}{3} L \]
for small \( \theta \)

\[ \delta = \frac{L}{3} y + k\delta \frac{2}{3} L \]
but \( y = \frac{2}{3} L \theta \)

\[ \theta = -k \left( \frac{2}{3} L \right)^2 \theta + k\delta \frac{2}{3} L \]
but \( k\delta = \frac{3}{4} Mg \)

\[ \theta = -k \left( \frac{2}{3} L \right)^2 \theta + \frac{1}{2} Mg L \]

\[ \text{Total torque: } \tau = -Mg \frac{L}{2} - k \left( \frac{2}{3} L \right)^2 \theta + \frac{1}{2} Mg L \]

\[ \tau = -k \left( \frac{2}{3} L \right)^2 \theta \]

\[ \dot{L} = \frac{\delta \theta}{dt} \]
\[ I \frac{d\omega}{dt} = -k \left( \frac{2}{3} L \right)^2 \theta \]
\[ \frac{dL^2}{dt^2} = -k \frac{4L}{3} \theta \]
\[ \frac{d\theta}{dt} = -4 \frac{k}{3m} \theta \Rightarrow \omega^2 = \frac{4}{3} \frac{k}{m} \]
\[ \omega = 2\pi f \Rightarrow f = \frac{1}{2\pi} \omega = \frac{1}{2\pi} \sqrt{\frac{4}{3} \frac{k}{m}} \]

\[ = \frac{1}{2\pi} \sqrt{\frac{4}{3} \frac{10^3}{3}} \]

\[ f = 5.8 \text{ Hz} \quad T = 0.17 \text{ s} \]