4.1 Criteria for evaluating optical imaging systems

4.1.A  Optics in different regimes
4.1.B  The process of imaging through an optical system. Wavefronts and limitations
4.1.C  Criteria for evaluating an optical imaging system

4.1.A Optics in different regimes

- The electromagnetic spectrum


Radiobroadcasts ($\lambda \sim 100$ m), microwaves ($\lambda \sim$ mm), visible ($\lambda \sim 500$ nm), x-ray ($\lambda \sim 1$ nm), gamma-rays ($\lambda < 10^{-3}$ nm).
Classification of the optical regimes according to the detection systems

Geometrical optics or ray optics
Classical theory of electromagnetic radiation
Photon picture

Geometrical optics or ray optics

a) The wavelengths $\lambda$ involved are small compared with the dimensions $d_{\text{apparatus}}$ of the apparatus involved: $\lambda << d_{\text{apparatus}}$.

b) The photon energies $h\nu$ are small compared with the coarse energy sensitivity $\Delta E_{\text{apparatus}}$ of the apparatus: $h\nu << \Delta E_{\text{apparatus}}$.

Under these approximations we can make rough approximation by a method called **geometrical optics**, which practically omits the wave character of the electromagnetic radiation. That is $\lambda$ is considered practically equal to zero, $\lambda / d_{\text{apparatus}} \rightarrow 0$.

Within this Geometrical Optics regime approximation:

*Light travels out of its source along straight lines or rays.*

Ray refers to a path along which light energy is transmitted from one point to another in an optical system.

Classical theory of electromagnetic radiation

a) The wavelengths $\lambda \sim d_{\text{apparatus}}$

   (A bit difficult to do it with visible light, but easier to implement with microwaves)

b) The photon energies $h\nu$ are still small compared with the energy sensitivity $\Delta E_{\text{apparatus}}$ of the apparatus ($h\nu << \Delta E_{\text{apparatus}}$).

In this case, a very useful approximation can be made considering **wave theories**, while still disregarding the quantum mechanics.
character of light. This is the realm of the classical theory of electromagnetic radiation.

**Photon picture**

- For very short wavelengths (right side of the spectrum shown above) \( \lambda \ll d_{\text{apparatus}} \).
- The photon energies \( h\nu \) are large compared with the energy sensitivity \( \Delta E \) of the apparatus \( (h\nu \gg \Delta E_{\text{apparatus}}) \).

This regime constitutes the realm of the photon picture of the light.
**4.1 B The process of imaging through an optical system.** Wavefronts and limitations

Let’s consider the task of imaging a point light source.

![Diagram of imaging process](image)

**Object** $P$  
**Optical system**  
**Image** $P'$

Wavefront: Locus of points such that each ray contacting a wavefront represents the same transit time of light from the source

An image process involves the modification of the *wavefront* $\Sigma$ by the optical system.
An ideal optical system would make all the rays from $P$ to reach the image point $P'$ at the same time.

This could be achieved by properly shaping the refracting surface.

$P$ and $P'$ are called “conjugated points”.
Limitations of an optical system
No perfect image

In a **Fourier analysis** of image formation, it turns out that the higher resolution information is harder to gather.

Realizing that the imaging process by optical system will have limitations, we would like to **develop a formalism** (a theoretical formalism) that will allow us to **evaluate how much “imperfect” is the real image** from the idealized one.
11.1 C Criteria for evaluating an optical imaging

- Criteria for obtaining a “perfect” image\(^1\) of a point light source

To develop such formalism, let’s apply the approach of considering first a simpler situation, and only then to address a more complicated one.

The simplest thing to image is a point light source, like the point \(P\) shown in the figure below.

We would like to obtain the image of the point light source \(P\) through a refractive surface \(S\) (a boundary that divides two regions of different indices of refraction.)

What would constitute a perfect image of \(P\)?

\(^1\) By “perfect” we mean the best it can be done after factoring out the inherent limitations imposed by diffraction effects (i.e. an image will always be non-perfect). Hence, in the context of geometrical optics, a “perfect” image is one that is just limited by diffraction (i.e. the system has managed to overcome other limitations, such as spherical aberrations, chromatic aberration,…).
We impose that the image of the point P should be another point; for example the point P' shown in the figure below. To obtain a perfect image of P, then, all the rays emanating from P should arrive to P'.

Notice, however, that in general not all the rays leaving the point P will arrive at P', unless the interface $S$ has the proper shape (one that ensures that each rays striking the surface bend the right angle so that it is redirected towards P').

Thus, our task of obtaining a prefect image is equivalent to find the proper geometrical shape of the surface $S$ such that the all the rays leaving from P and hitting the surface deviates towards P'.

(2a)

On the other hand, keeping into account that light indeed has a wave nature, we impose the additional requirement that
all the rays that emanate from P arrive at P’ at the same time (otherwise, they could interfere destructively and thus spoil the image formation.)

These requirement will be included in the context of the Fermat’s principle of least time or, more generally, the variational principle.

- **The Fermat’s Principle of Least Time**

  It turn out, time ago in 1650, Fermat enunciated the principle that light, when going from one point to another, follows the particular path for which the time of flight is minimum (compare to the time it would take if another path were taken.) This principle will be described in more detail in the following sections.

  We will adopt this principle given its generality and power for making perdictions. In particular we will use it here as a tool to guide us in the design of devices that produce perfect images (in a manner that we will address below).

  Indeed, using this principle:
  a) we will be able to find the path followed by the light when it crosses interfaces (i.e. Snell’s law); and
  b) we will be able to find the shape of the interface $S$ (alluded in 2a above) necessary for obtaining a “perfect” image.
Outline for the next sections and lectures

Imaging through perfect surfaces

In the following sections of Lecture-11 we will describe the Fermat principle in some detail, and use it to derive the laws that govern how rays are refracted and reflected at interfaces (Snell’s law.) The Fermat principle will be described in the context of the “Variational Principle,” which has far more reaching consequences (even beyond the field of Optics.)

In Lecture-12, we will invoke the Fermat’s principle in order to find the actual shape an interface \( S \) must have in order to obtain the perfect image of a point.

Non perfect imaging surfaces

Anticipating that perfect imaging surfaces \( S \) may, in general, be too difficult to fabricate, simpler surfaces, such as spherical surfaces, will be considered. The imaging through spherical surfaces will be covered in Lecture-13.

As a consequence of using spherical boundary surfaces (and not the aspherical surface \( S \)), the rays going from \( P \) to \( P' \) will not arrive at the same time. The time delay between two particular rays will provide a measurement of the “imperfection” of the spherical imaging surface. It is in that sense that the Fermat
principle is considered our standard theoretical principle to evaluate the quality of an imaging system.

In summary:
• The imaging though an aspherical surface $S$ (one that satisfies requirements 2a and 2b mentioned above) will be assumed to be our referenced “perfect” imaging tool. (We know this connotation of perfection is not true, since diffractions effects, at the need, spoils this perfection). Anyway, aside from diffraction, this is the best we can do (within this geometrical approximation).
• But since these aspherical surfaces for perfect imaging are hard to fabricate, alternative spherical surface will be used instead. Therefore an inherent degree of “imperfection” will be associated to an imaging process though spherical surfaces.
• To measure the degree of imperfection the variational principle will be invoked: A given ray will not be a perfect imaging ray if it does not arrive at the same time than the other more “perfect rays”. The larger the difference, the worst. We call “perfect rays” those that arrive from P to P’ at the same time.