Relationship between the \( E \) and \( B \) fields

Case: The fields between two metallic plates

Stationary waves. The Heinrich Hertz experiment

\[
\begin{align*}
\vec{E}_i &= E_m \cos(kx - \omega t) \hat{y} \\
\vec{E}_r &= E_m \cos(kx + \omega t + \varphi_R) \hat{y} \\
\end{align*}
\]

\( \varphi_R \) is included to account for possible phase shift upon reflection.

Using the identities

\[
\begin{align*}
\cos(a+b) &= \cos(a) \cos(b) - \sin(a) \sin(b) \\
\cos(a-b) &= \cos(a) \cos(b) + \sin(a) \sin(b) \\
\end{align*}
\]

gives

\[
\begin{align*}
\cos(a+b) + \cos(a-b) &= 2 \cos(a) \cos(b) \\
\cos(A) + \cos(B) &= 2 \cos[(A+B)/2] \cos[(A-B)/2] \\
\end{align*}
\]

\[
\vec{E}_{\text{total}}(x,t) = \vec{E}_i + \vec{E}_r = 2 \cos\left[kx + \frac{\varphi_R}{2}\right] \cos\left[\omega t + \frac{\varphi_R}{2}\right] \hat{y}
\]
We require the total electric field to be zero at the metal plate (i.e. at $x=0$) and at all time.

At $x=0$:

\[
\vec{E}_{total}(0,t) = 2 \cos\left[\frac{\varphi_R}{2}\right] \cos\left[\omega t + \frac{\varphi_R}{2}\right] \hat{y} = 0
\]

This requires $\varphi_R = \pi$

Thus,

\[
\vec{E}_i = E_m \cos(kx - \omega t) \hat{y}
\]

\[
\vec{E}_r = E_m \cos(kx + \omega t + \pi) \hat{y} = -E_m \cos(kx + \omega t) \hat{y}
\]

\[
\vec{E}_{total}(x,t) = \vec{E}_i + \vec{E}_r = 2 \sin(kx) \sin(\omega t) \hat{y}
\]

On the other hand, before we also know that the ME require,

\[
\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}
\]

From which one obtains,

\[
2k \cos(kx) \sin(\omega t) = -\frac{\partial B_z}{\partial t}
\]

\[
B_z = 2 \frac{k}{\omega} \cos(kx) \cos(\omega t)
\]

\[
\vec{B}_{total}(x,t) = 2 \cos(kx) \cos(\omega t) \hat{z}
\]
Notice $E_{\text{total}}$ and $B_{\text{total}}$ will have nodes and antinodes along the $X$ axis.
antinode A

node B

\[ B = 0 \]

\[ E \] will produce a spark

\[ B(t) \]

electromotive force along the wire
By measuring the distance between 2 antinodes, Hertz was able to figure out $\lambda/2$. Since he knew the frequency $\omega$ of the oscillator (transformer), he could calculate the velocity $\lambda f$ ($f = \frac{\omega}{2\pi}$) of the electromagnetic waves. $v = 300,000 \text{ km/s}$
Case: The fields in the free and unbounded space
Relationship between the electric field and magnetic field amplitudes

Let's consider an harmonic plane wave

\[ E = E_m \cos(kx - \omega t) \]
\[ B = B_m \cos(kx - \omega t) \]

\[ \frac{\partial E}{\partial x} = -k E_m \sin(kx - \omega t) \]
\[ \frac{\partial B}{\partial t} = \omega B_m \sin(kx - \omega t) \]

Equation 0 implies \[ k E_m = \omega B_m \] or \[ B_m = \frac{E_m}{c} \]

We will leave for the HW assignment to demonstrate that \[ \vec{E} \] and \[ \vec{B} \] are perpendicular to one another. \[ \vec{E} \times \vec{B} \] points in the direction where the wave propagates toward to.

indicates vector product