6.1 INTRODUCTION

The phenomena of refraction and refraction of light at a plane surface separating two media of different dielectric properties can be classified into two aspects: the kinematics and the dynamic properties.

**Kinematics properties**

These include the relationship that exists between the angles of incidence \( \theta_{\text{incid}} \), reflection \( \theta_{\text{reflection}} \), and refraction \( \theta_{\text{refraction}} \)

1. \( \theta_{\text{incid}} = \theta_{\text{reflection}} \)
2. \( n_i \sin \theta_{\text{incid}} = n_r \sin \theta_{\text{reflection}} \)

These two relations result from the fact that, “there are boundary conditions to be satisfied; but they do not depend on the detailed nature of those boundary conditions.” Ref. J. Jackson, “Classical Electrodynamics.”

For example, let’s assume that a particular boundary condition has the form

\[
A \vec{E}_{\text{inc}}(\vec{x},t) + B \vec{E}_{\text{reflec}}(\vec{x},t) + C \vec{E}_{\text{transm}}(\vec{x},t) = \vec{0} \quad \text{at } z=0
\]

which has to be to be satisfied at any position \( \vec{x} \) on the interface (see Fig. above,) and at any time \( t \).
The particular values of $A$, $B$ and $C$ will depend on the particular polarization of the fields we are dealing with (s-polarization or $p$-polarization, which will be defined below.) $A$, $B$, and $C$ will then be determined by the Maxwell Equations. But, whatever the relationship among $A$, $B$ and $C$ is, that does not affect the relations shown in $i)$ and $ii)$ above; those results are obtained (as we will see in the next section) simply from the fact that there are boundary conditions that need to be satisfied.

**Dynamic properties**

These include,

- $iii)$ The intensities of the reflected and transmitted radiation
- $iv)$ Phase changes and polarization

These properties depend on the particular nature of the boundary conditions. That is, they depend on the particular values of $A$, $B$, and $C$ that appear in expression (1).

**PROPELLATING PLANE WAVES IN A DIELECTRIC MATERIAL**

- To formally derive the kinematics and dynamics properties mentioned above, we will consider the propagation of plane waves.

In each dielectric medium, light propagation is governed by the wave equation,

$$\nabla^2 \begin{bmatrix} \tilde{E} \\ \tilde{B} \end{bmatrix} - \frac{\varepsilon / \varepsilon_0}{c^2} \frac{\partial^2}{\partial t^2} \begin{bmatrix} \tilde{E} \\ \tilde{B} \end{bmatrix} = 0$$

Or equivalently,

$$\nabla^2 \begin{bmatrix} \tilde{E} \\ \tilde{B} \end{bmatrix} - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \begin{bmatrix} \tilde{E} \\ \tilde{B} \end{bmatrix} = 0 \tag{1}$$

where the dielectric constant $\varepsilon / \varepsilon_0 = n^2$ will be assumed to be, in general, a complex number.

(We derived the above expressions in Section 2.2B; expression (17) of that section.)

- Equation (1) admits plane waves as solutions,

$$\tilde{E}(\vec{r}, t) = \tilde{E}_o e^{i(k \cdot \vec{r} - \omega t)}$$

$$\tilde{B}(\vec{r}, t) = \tilde{B}_o e^{i(k \cdot \vec{r} - \omega t)} \tag{2}$$

provided that,

$$k^2 = \frac{\omega^2}{c^2} n^2.$$
Since \( \omega/c = 2\pi f/c = 2\pi /\lambda_o \) (where \( \lambda_o \) is the wavelength of light in vacuum),

\[
k^2 = \left( \frac{2\pi}{\lambda} \right)^2 \tag{1}
\]

But to simplify the notation, in this chapter we will use simply \( \lambda \) for the wavelength of light in vacuum, instead of \( \lambda_o \). That is,

\[
k^2 = \left( \frac{2\pi}{\lambda} \right)^2 \tag{3}
\]

We emphasize that the solutions in (2) are valid even when \( n^2 \) (hence \( k^2 \)) is complex.

- On the other hand, the Maxwell equation \( \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \mathbf{0} \), when applied to the plane waves in (2), implies \( i\vec{k} \times \vec{E}_o - i\omega \vec{B}_o = \mathbf{0} \). That is,

\[
\vec{k} \times \vec{E}_o = \omega \vec{B}_o \nonumber
\]

Equivalently,

\[
\frac{\vec{k} \times \vec{E}_o}{\omega} = \frac{\vec{B}_o}{k} \nonumber
\]

\( \omega/k \) is the phase velocity \( v_{\text{phase}} = 1/\sqrt{\mu\varepsilon} \), which gives

\[
\frac{\vec{k} \times \vec{E}_o}{k} = \frac{1}{\sqrt{\mu\varepsilon}} \vec{B}_o ;
\]

\[
\vec{B}_o = \sqrt{\mu\varepsilon} \frac{\vec{k} \times \vec{E}_o}{k} \tag{4}
\]

- Electromagnetic waves propagate energy

\[
\vec{S} = \vec{E} \times \frac{1}{\mu} \vec{B} \quad \text{energy flow per unit area per second}
\]

where \( \mu \) is the magnetic permeability of the medium.

For harmonic fields, like the one in (2), it turns out,

\[
I = \langle \vec{S} \rangle = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} |\vec{E}_o|^2 \frac{\vec{k}}{k} \quad \text{time-average energy per unit area} \tag{5}
\]

\( I \) is called the light intensity.

Equivalent forms of (5) are,

\[
I = \langle \vec{S} \rangle = \frac{\varepsilon}{2} |\vec{E}_o|^2 \frac{\vec{k}}{k} = \frac{\varepsilon c}{2n} |\vec{E}_o|^2 \frac{\vec{k}}{k}
\]
\[ I = \langle \tilde{S} \rangle = \frac{1}{2\mu v} |\tilde{E}_0|^2 \frac{\tilde{k}}{k} = \frac{n}{2\mu c} |\tilde{E}_0|^2 \frac{\tilde{k}}{k} \]

where \( n \) is the index of refraction of the medium.

Terminology that is used to describe reflection and refraction at a dielectric-dielectric interface

We adapt the solutions given in (2) to setup the problem of reflections and refraction at an interface.

Three general plane waves are considered,

\[ \text{Incident wave} \quad \tilde{E}_i = \tilde{E}_{oi} e^{j(\tilde{k}_i \cdot \tilde{r} - \omega_i t)} \]

\[ \text{Reflected wave} \quad \tilde{E}_r = \tilde{E}_{or} e^{j(\tilde{k}_r \cdot \tilde{r} - \omega_r t)} \] \hspace{1cm} (5)

\[ \text{Transmitted wave} \quad \tilde{E}_t = \tilde{E}_{ot} e^{j(\tilde{k}_t \cdot \tilde{r} - \omega_t t)} \]

Or equivalently, if we want to manipulate just real variables

\[ \tilde{E}_i = \tilde{E}_{oi} \cos(\tilde{k}_i \cdot \tilde{r} - \omega_i t) \]
\[ \tilde{E}_r = \tilde{E}_{or} \cos(\tilde{k}_r \cdot \tilde{r} - \omega_r t) \] \hspace{1cm} (6)
\[ \tilde{E}_t = \tilde{E}_{ot} \cos(\tilde{k}_t \cdot \tilde{r} - \omega_t t) \]
We need to find out the relationship among the $\vec{k}$ vectors (so far we are assuming they have arbitrary orientations,) as well as the relationship among the amplitudes $\vec{E}_{oi}$, $\vec{E}_{or}$, and $\vec{E}_{ot}$.

We write the corresponding $\vec{k}$ vectors more explicitly,

$$\vec{k}_i = k_{ix} + k_{iy} + k_{iz}$$
$$\vec{k}_r = k_{rx} + k_{ry} + k_{rz}$$
$$\vec{k}_t = k_{tx} + k_{ty} + k_{tz}$$

According to (3), and considering that $\vec{k}_i$ and $\vec{k}_r$ are in the same medium, we have,

$$|\vec{k}_i| = |\vec{k}_r| = \frac{2\pi}{(\lambda / n_i)} = \frac{2\pi}{\lambda} n_i$$
$$|\vec{k}_t| = \frac{2\pi}{(\lambda / n_t)} = \frac{2\pi}{\lambda} n_t$$

$n_i$ and $n_t$ are the indices of refraction of the two media, respectively. $\lambda$ is the wavelength of the incident light when in vacuum.