NEGATIVE FEEDBACK and APPLICATIONS

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I. PURPOSE:
To use various types of negative feedback, using operational amplifiers, to build a) gain-controlled amplifier, non-inverted amplifier, integrator, differentiator; and b) active filters.
Students must i) provide an analysis of the circuit in the time domain, and ii) evaluate the corresponding transfer function (Laplace domain analysis) of the circuits.

II. THEORETICAL CONSIDERATIONS

FEEDBACK
In control systems, feedback consists in comparing the output of the system with the desired output and making a correction accordingly.¹

II.1 Negative feedback
Negative feedback is the process of coupling a portion of the output back into the input, as a way to cancel part of the input. This process, it turns out, has the effect of reducing the gain of the amplifier, but, in exchange, it improves other characteristics including freedom from distortion and nonlinearity, flatness in the frequency response, and predictability. In fact, as more negative feedback is used, the resultant amplifier’s characteristics becomes less dependent on the characteristics of the original open-loop amplifier.

Caution: We have learned from our RC filter experiment that a phase lag exists between the input and output voltages, which increases as more components are added into the circuit. If a negative feedback-loop circuit were to accumulate large enough phase lag (i.e. greater than 180°), then positive feedback occurs (the circuit ends up being an oscillator.)

![Operational amplifier](image)

Fig. 1 Operational amplifier.

II.2 OPERATIONAL AMPLIFIERS WITH “INFINITE and FINITE GAIN
- See additional information contained in the file “Operational Amplifiers (background)” posted on the website of this course.
Before performing the experiments listed below, review the data sheet of the op amp you are using. In particular, check the slew rate (it is 0.3 V/μs for the LM358AP). This parameter (that tell you how fast the op-amp responds to an input signal) will allow you to estimate the frequency bandwidth response of your circuit.

II.2.A The concept of open loop gain and close loop gain

**Fig. 2** Typical open loop voltage gain $A$ as a function of frequency.

When integrated with an external network, the gain of the network is less than the open-loop gain of the amplifier. (Even though the gain is lower, the performance in terms of stability is much better).

**Fig. 3** Open loop $A$ and close loop $A_{CL}$ voltage gain as a function of frequency. Notice in the graph that $A_{CL} < A$.

II.2.B Op-amps with infinite open loop gain:

The Golden Rules of Operational Amplifiers

When implemented as part of a negative feedback external network, the behavior of the op amp can be predicted (for many practical purposes) based on two simple rules:

1. **Rule 1:**
   
   $A_{CL} = R_f / R_o$

2. **Rule 2:**
   
   $f_T = 1 / (2 \pi R_f C_r)$
**Rule I**  The output attempts to do whatever is necessary as to produce that the external feedback brings the differential input voltage close to zero \((v_+ - v_- \approx 0)\).

\[ (1) \]

**Rule II**  The inputs draw no current \((i_-, i_+ \approx 0)\).

Basically we are saying that the open loop gains A is infinite. This is of course an abstraction, but allows obtaining a quick grasp of the functioning of many widely used circuits (as we will see in the experimental section below).

III.  **EXPERIMENTAL CONSIDERATIONS**

Applications of negative feedback with operational amplifiers

III.A  Amplifiers Circuits

III.B  Active Filters

III.1  Amplifiers Circuits

III.1.A  The Inverting Amplifier

Since \(v_+\) is grounded, then *Rule I* implies \(v_- = 0\). Accordingly,

- the current through \(R_o\) is equal to \(I_o = V_{in} / R_o\); and
- the current through \(R_f\) is then equal to \(I_f = -V_{out} / R_f\).

Since no current flows into the op amp inputs (*Rule II*) we should have

\[ I_o = I_f \]

that is,

\[ V_{in} / R_o = -V_{out} / R_f \]

or simply,

\[ V_{out} = -\frac{R_f}{R_o} V_{in} \]

\[ (2) \]

**TASKS:**

a)  Construct the circuit shown in Fig. 4, and verify that indeed the voltage gain is equal to \(R_f / R_o\).

b)  Testing the operational amplifier in the frequency domain. Using a sinusoidal wave (~ 50 mV amplitude) as an input, make two Bode plots, corresponding to two different gains (10 and 100 for example).

Evaluate the corresponding \(f_{3dB}\) for these two cases. (i.e. evaluate if the \(f_{3dB}\) value depends on the gain.)

c)  Testing the operational amplifier in the frequency domain. This step is to get familiar with the concept of slew-rate [see also the “slew-rate” section in the file “Operational amplifier (background material)” posted online in the website of this course]
For a given value of the slew-rate $S$, we expect the maximum amplitude at which a sine-wave output of frequency $f$ remains undistorted to be given by $S = 2\pi f B_{\text{max}}$. That is, the amplitude $B$ of a sine-wave $B \sin(\omega t)$ must fulfill the condition

$$B \leq \frac{S}{2\pi f}$$  \hspace{1cm} (3)

**c1)** Using first a square-wave input of 50 mV amplitude, increase gradually the frequency until you start noticing that the output signal does not follow exactly the input. You are this way testing the limitation of the operational amplifier in the time domain (rather than in the frequency domain, as in part $b$ above).

**c2)** Change you input to a sinusoidal wave, and use a fixed frequency $f$ that is lower than, but close to, the 3dB found in part $b$ above. Check the amplitude of the output signal has the expected value (for the given gain, 10 or 100, you are using). Now take different (increasing) values of the input voltage amplitude $B$. Verify if the amplitude of the output-signal deviates very much from its expected value when the input-signal amplitude grows too large. Check if this deviation start happening when $B$ starts exceeding the value $B_{\text{max}}=S/2\pi f$ (where $S$ is the slew rate you obtain from the data sheet of the op-amp). Make this evaluation for the two cases, gain 10 and 100.

**d)** We analyzed above the functioning of the inverted amplifier circuit in the time domain. This time, instead, we ask you to also analyze the circuit in the Laplace domain; evaluate the transfer function of this circuit.

Suggestion: Proceed with a time-domain analysis first, and then evaluate the corresponding Laplace transformation of the quantities involved.
III.1.B The Noninverting Amplifier

*Rule 1* implies $v_+ = V_{in}$

At the same time, $v_+$ is part of a voltage divider: $v_+ = [V_{out} / (R_1 + R_2)] R_3$

Equating these two expressions, we obtain,

$$V_{out} = \left[ 1 + \frac{R_2}{R_1} \right] V_{in} \quad (4)$$

**Implementation:**

a) Construct the circuit shown in Fig. 6, whose input is a DC voltage, and verify that the voltage gain is indeed equal to $1 + \left( \frac{R_2}{R_1} \right)$. Using a square wave (~50 mV amplitude) as an input, make two Bode plots, corresponding to two different gains (10 and 100 for example). Check if the $f_{3dB}$ value depends on the gain.

b) Analyze the circuit in the Laplace domain; evaluate the transfer function of this circuit.

   Suggestion: Proceed with a time-domain analysis first, and then evaluate the corresponding Laplace transformation of the quantities involved.

III.1.C Differential Amplifier  

*(SKIP THIS SECTION IN 2004)*
Implementation:

a) Apply the golden rules to demonstrate that the output voltage of the circuit shown in Fig. 5 is given by,

\[ V_{out} = \frac{R_2}{R_1} (V_2 - V_1) \] (5)

b) Build the circuit shown in Fig. 7 and verify if the output voltage varies according to the expression given in part a) above.

![Differential amplifier](image)

**Fig. 7** Differential amplifier.

III.1.D Integrator

Op amps allow to make integrators without the restriction that \( V_{out} < V_{out} \). (as required when using only R and C components.)

Since \( V_+ \) is grounded, the input \( V_- \) acts as a virtual ground. The current \( I_R \) passing through the resistor is then given by,

\[ I_R = \frac{V_{in}}{R} \]

The current through the capacitor is given by,

\[ I_C = \frac{d}{dt}(q) = \frac{d}{dt}(0 - V_{out})C \]

\[ = -C \frac{dV_{out}}{dt} \]

Rule II implies that \( I_R = I_C \),

\[ \frac{V_{in}}{R} = -C \frac{dV_{out}}{dt} \]

This implies,

\[ V_{out}(t) = -\frac{1}{RC} \int^t_{t'} V_{int}(t')dt' \] (6)
Implementation:

a) Implement an integrator circuit shown in Fig. 8. Since charging effects can cause serious offsets, a parallel resistor $R_p$ may be needed (to prevent any long term voltage shift at the input). Try different values for $R_p$. (100KΩ, 1 MΩ). See Fig. 9.

b) Test your circuit using a square signal of 1 kHz at the input. Investigate the effect of changing the various parameters.

c) Analyze the circuit in the Laplace domain; evaluate the transfer function of this circuit.

   Suggestion: Proceed with a time-domain analysis first, and then evaluate the corresponding Laplace transformation of the quantities involved.

III.1.E Differentiator

The circuitry is similar to the integrator but with the R and C reversed.

The current through the capacitor is given by,

$$I_c = \frac{d}{dt}(q) = \frac{d}{dt}(V_{int}C) = C \frac{dV_{in}}{dt}$$

The current through the resistor is given by,
\[ I_R = \frac{0-V_{out}}{R} \]

Rule II implies that \( I_R = I_C \),

\[ C \frac{dV_{in}}{dt} = -\frac{V_{out}}{R} \]

This implies,

\[ V_{out}(t) = -RC \frac{dV_{in}}{dt} \]  \hspace{1cm} (7)

**Implementation:**

a) Implement a differentiator circuit. Test the circuit with triangle waves at the input.

**Fig. 10 Differentiator**

b) We analyzed above the functioning of the circuit in the time domain. This time, instead, analyze the circuit in the Laplace domain; evaluate the transfer function of this circuit.

**III.2 Active Filters**

**III.2.A Exploiting the low output impedance and high input impedance of op-amps**

In a previous laboratory you worked with two first-order low pass filters connected in a cascade arrangement, as shown in Fig. 11.

**Fig. 11 Two low-pass RC filters.**

The output voltage of this circuit is given by,

\[ v_{out} = v_{in} \frac{z_C}{(R + z_C)} \frac{z_C}{(R + z_C)} \left[ \frac{1}{1 + \frac{R z_C}{(R + z_C)^2}} \right] \]

where \( z_C \) is the capacitor impedance.
Notice, we could not have stated that $v_{out}$ was simply the product of two voltage dividers expressions like \[ \frac{Z_C}{(R + Z_C)} \cdot \frac{Z_C}{(R + Z_C)} \] because of the “loading” effects of the second simple low-pass filter over the first simple low-pass filter.

**TASK:** Using expression (8) prove that,

\[ v_{out} = \frac{1}{(j\omega RC + 1)^2 + j\omega RC} v_{in} \]  \hspace{1cm} (9)

One way to get around the loading effect one circuit stage exerts on another is to use operational amplifiers in a “follower” configuration. Notice, it is simply a non-inverting amplifier, as shown in Fig. 6 above with $R_2=0$ and $R_2=\infty$).

**TASKS:** Build the circuit shown in Fig. 12.

Make a Bode plot and identify the corresponding $f_{3dB}$ frequency. Compare the result with the one corresponding to the circuit in Fig. 11 (obtained in your previous lab-4).

Evaluate $v_{out}$ using an analysis in the time domain (use the op-amp golden rules).

Evaluate the corresponding transfer function (analysis in the Laplace domain).

In both cases justify all your steps.

**Fig. 12** Low pass filters with a decoupling amplifier.

**Hint:** The answers you should arrive are the following,

\[ \frac{v_{out}}{v_{in}} = \frac{Z_C}{(R + Z_C)} \cdot \frac{Z_C}{(R + Z_C)} \]  \hspace{1cm} (10)

In the time domain $Z_C = 1/(j\omega C)$; hence,

\[ \frac{v_{out}(t)}{v_{in}(t)} = \frac{1}{(1 + j\omega RC)} \cdot \frac{1}{(1 + j\omega RC)} \]  \hspace{1cm} (11)

In the time domain $Z_C = 1/(C\cdot s)$; hence,

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{(1 + RC\cdot s)} \cdot \frac{1}{(1 + RC\cdot s)} \]  \hspace{1cm} (12)
III.2. B Active pass filters: Filters with steeper roll-off

Simple RC filters produce gentle low low-pass gain characteristics with a roll-off of 6dB/octave at frequencies well beyond the -3dB frequency. Such filters are sufficient for many multiple purposes. Often, however, filters with flatter pass-bands and steeper roll-off are needed.

One solution may consists of cascading many RC filters, using buffer amplifiers (as indicated in Fig. 12) to avoid loading effects. It turns out, such an approach produces indeed steeper roll-off, but the curved “3dB knee” in the gain vs frequency response does not disappear.

On the other hand, filters made with inductors and capacitors can have very sharp responses. In fact, by including inductors in the design, it is possible to create filters with any desired flatness of passband, combined with roll-off steepness. The only problem is that inductors as circuit elements are often bulky and expensive (not to mention having significant series resistance. Hence the task becomes to design “inductorless filters” with the characteristics of ideal RLC filters.

Solution: Using op-amps as part of the filter design, it is possible to synthesize any RLC filter characteristic without using inductors.

A specific example of this statement is provided in the next example, where you are asked to compare the gain, as well as the transfer functions, corresponding to a RLC passive filter and a active RC filter.

TASKS: Consider the two circuits shown in Fig. 13.

a) For the passive RLC filter:

a1) Using the method of complex impedance in the time domain, show that

\[
\frac{v_{\text{out}}(t)}{v_{\text{in}}(t)} = \frac{1}{-\omega^2 LC + j\omega RC + 1}
\]  

(13)

a2) Using the method of complex impedance in the Laplace domain, show that,

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{LC s^2 + RC s + 1}
\]  

(14)

Hint: Use the expressions, in the Laplace domain, for the individual impedances of the R, C and L elements, as given in the “Transfer Function” notes posted in the website of this course.

![RLC passive filters](image1.png) ![Active RC pass filter](image2.png)

**Fig. 13 Left:** RLC passive filters. **Right:** Active RC pass filter.
b) For the active RC filter shown in Fig. 13:

b1) Show that
\[
\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{z_C}{(R + z_C)} \frac{z_C}{(R + z_C)}
\]

\[ (15) \]

b2) Show that in the time domain, where \( z_C = 1/(j\omega C) \), the expression above becomes,
\[
\frac{v_{\text{out}}(t)}{v_{\text{in}}(t)} = \frac{1}{1 + j\omega RC} \frac{1}{1 + j\omega RC}
\]

\[ (16) \]

Show that in the Laplace domain, where \( z_C = 1/(C \cdot s) \), the expression above becomes,
\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{(1 + RC \cdot s)} \frac{1}{(1 + RC \cdot s)} = \frac{1}{(RC)^2 \cdot s^2 + 2RC \cdot s + 1}
\]

\[ (17) \]

Hint: Use the expressions, in the Laplace domain, for the individual impedances of the \( R \), \( C \) and \( L \) elements, as given in the “Transfer Function” notes posted in the website of this course.

c) Compare the transfer functions for the passive \( RLC \) circuit (given by expression (14)) and the transfer function for the active \( RC \) filter (given by expression (17)). Include your comments on whether this active \( RC \) pass filter will behave similar to a passive \( RLC \) circuit.
2 (Page 177) Horowitz and Hills,