I. PURPOSE:

This laboratory session pursues two main objectives.

First, to build a low-pass RC filter and measure the output voltage (magnitude and phase) as a function of its frequency. A log-log plot (Bode plot) of the output signal vs frequency will help familiarize with the concepts of “3dB breaking point” and “decrease of output levels per octave and per decade.”

Second, familiarize with the functioning of the operational amplifiers, and its application in comparator circuits. Comparators that uses no feedback and a Schmitt Trigger comparator (that uses positive feedback) will be built.

II. THEORETICAL CONSIDERATIONS: LOW PASS FILTERS

II.1 The scale of decibels

Comparison of signal amplitudes
Comparison of Power levels

II.2 Single-stage low-pass filter

Output voltage (magnitude and phase)

II.3 Bode-plots and the 3dB breakpoint

II.4 Voltage drop at high frequencies (in decibels) in single low-pass filter

II.4A Roll off per octave

II.4B Roll off per decade

II.5 Dual-stage low-pass filter

II.1. The scale of decibels

Comparison of signal amplitudes

Let’s consider two signals of amplitudes $A_1$ and $A_2$ respectively.

The ratio of these two signals is: $\frac{A_1}{A_2}$

Because we often we deal with ratios that change by many order of magnitude during a testing procedure, the decibel scale is frequently used. By definition,

$$\left( \frac{A_1}{A_2} \right)_{dB} \equiv 20 \log_{10} \frac{A_1}{A_2}$$

(1)

$log_{10} 1 = 0$, $log_{10} 2 = 0.3010$, $log_{10} 3 = 0.477$, $log_{10} 5 = 0.6989$

$log_{10} A'' = n log_{10} A$, $log_{10} AB = log_{10} A + log_{10} B$
Example: \( \frac{A_1}{A_2} = 2 \) is equivalent to +6 dB

because \( \left( \frac{A_1}{A_2} \right)_{dB} = 20 \log_{10} 2 = 6 \)

We say, \( A_1 \) is +6 dB relative to \( A_2 \).

Example: A signal \( A_1 \) 10 times as large as \( A_2 \) is +20 dB
A signal \( B_1 \) one-tenth as large as \( B_2 \) is -20 dB

Comparison of Power levels
If \( P_1 \) and \( P_2 \) represent the power of two signal levels, their ratio in decibels is defined by,

\[
\left( \frac{P_1}{P_2} \right)_{dB} \equiv 10 \log_{10} \frac{P_1}{P_2}
\]

II.2 Low–pass filter

\[ V_{\text{in}} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V_{\text{out}} \]

\[ R \]

\[ C \]

Fig.1 A low pass filter is obtained with a resistor and a capacitor connected in series while the output is taken across the capacitor.

Output voltage (magnitude and phase)
In Fig.1, assuming

a zero output impedance of the driving source, (which provides \( V_{\text{in}} \)) and

an infinite input impedance of the loading device. (to which \( V_{\text{out}} \) is connected,)

we obtain,

\[
V_{\text{out}} = \frac{V_{\text{in}}}{R + Z_C} Z_C = \frac{V_{\text{in}}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1} V_{\text{in}}
\]

\[
V_{\text{out}} = \frac{1}{\sqrt{(\omega RC)^2 + 1}} e^{j\varphi} V_{\text{in}} \quad \text{where} \quad \varphi = \tan^{-1}(-\omega RC)
\]

That is, the output voltage \( V_{\text{out}} \) lags the input voltage \( V_{\text{in}} \).

The ratio of the input and output voltage magnitude is given by,

\[
\left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}}
\]

(4)
II.3 Bode-plots and the 3dB breakpoint

RC is the characteristic time response of the low-pass RC circuit.

At $\omega = 1/RC$:

- The ratio $|$ $v_{out}$/$v_{in}$ $|$ drops to $1/\sqrt{2} = 0.7$
  
  In the decibels scale, this ratio is equal to $20 \log_{10}(0.7) = -3$; that is $|$ $v_{out}$/$v_{in}$ $|$ $dB = -3$ dB

- The change in phase is 45°.

Fig. 2 Left: Frequency response of a low pass filter. Right: The right figure displays the same data but in a logarithmic scale.

In general (no just for an RC filter)

*The frequency at which the output voltage drops by -3 dB is referred to as the “-3 dB breakpoint” of a filter (or of any circuit that behaves as a filter.).*

II.4 Voltage drop at high frequencies (in decibels)

Let's express the ratio of voltages in decibels. From expression (4) we obtain,

$$\left| \frac{v_{out}}{v_{in}} \right|_{dB} = 20 \log_{10} \left| \frac{v_{out}}{v_{in}} \right| = 20 \log_{10} \frac{1}{\sqrt{(\omega RC)^2 + 1}} = -10 \log_{10} (\omega RC)^2 + 1$$

At high frequencies, $\omega \gg 1/RC$,

$$\left| \frac{v_{out}}{v_{in}} \right|_{dB} = 20 \log_{10} \left| \frac{v_{out}}{v_{in}} \right| \xrightarrow{\text{large } \omega} -20 \log_{10} (\omega RC)$$

That is,

$$\left| \frac{v_{out}}{v_{in}} \right|_{dB} \xrightarrow{\text{for large } \omega} -20 \log_{10}(\omega) - 20 \log_{10}(RC)$$ (5)
We expect, then, that at large frequencies (i.e. for $\omega \gg 1/RC$), a plot of $\frac{V_{out}}{V_{in}}$ vs $\log_{10}(\omega)$ should linearly decrease with a slope of 20. This is in fact verified in the figure below.

**Fig. 3** Linear decrease of the output signal (in decibels) at high frequencies.

### II.4A Roll-off per octave

We are familiar with the fact that in music, in one octave the frequency is doubled. (6)

Similarly in our case of electrical signal changes (as described above,) we can ask ourselves how much does the voltage ratio in expression (5) changes (in decibels) per octave?

**Answer:**

In one octave (a range where the frequency doubles) the horizontal axis in Fig. 4 changes by

$$[\log_{10}(2\omega) - \log_{10}(\omega)] = \log_{10}(2) = 0.3010.$$ 

Since the slope is -20, a change in the horizontal axis by 0.30 will give a vertical change equal to $-20 \times 0.30 = -6$ dB

Accordingly, we say:

*In a simple RC low-pass filter the output voltage drops -6 dB per octave*  

(7)

**Fig. 4** Roll-off pr octave. Left: Linear drop of the output/input voltage ration (in decibels)
at high frequencies in a logarithmic scale. **Right:** Same data but in a zoomed-in scale.

Equivalently,

*in a simple RC low-pass filter the output voltage decreases by a factor of 2 per octave.*

(This follows from the fact that for \((A_1/A_2) = 1/2\) one has \((A_1/A_2)_{dB} = 20 \log_{10}(1/2) = -6\) dB.)

II.4B Decibels per decade

In a *decade*, the frequency changes by a factor of 10. (8)

How much does the voltage ratio in expression (5) changes (in decibels) per decade?

Answer:

In one decade (a range where the frequency changes by a factor of 10) the horizontal axis in Fig. 5 changes by

\[
\left( \log_{10}(10\omega) - \log_{10}(\omega) \right) = \log_{10}(10) = 1.
\]

Since the slope is -20, a change of 1 in the horizontal axis will correspond to a vertical axis equal to -20 x 1 = -20 dB

Accordingly, we say:

*In a RC low-pass filter the output voltage drops -20 dB per decade* (9)

---

**Fig. 5** Response from a single-stage low-pass filter
Equivalently, in a simple RC low-pass filter the output voltage decreases by a factor of 10 per decade.

(This follows from the fact that for \( A_1 / A_2 = 1/10 \) one has \( (A_1 / A_2)_{dB} = 20 \log_{10}(1/10) = -20 \text{ dB} \).)

II.5 Dual-stage low–pass filter

**TASK:** Derive an expression for \( v_{out} \) in terms of \( v_{in} \).

![Fig. 6 Two-stage low-pass filter.](image)

III. EXPERIMENTAL CONSIDERATIONS

III.1 RC FILTER

III.1A Single-stage filter

Implement a RC Low-Pass filter. (You may use \( C = 0.22 \text{ μF} \) and \( R = 1 \text{ KΩ} \); of course, feel free to try other values.) Use a signal generator to provide a sinusoidal input signal \( v_{in} \) of \( \sim 400 \text{ mV} \) amplitude, and check with the oscilloscope whether the output voltage leads or lags the input voltage. Suggestion: Monitor \( v_{in} \) and \( v_{out} \) in the oscilloscope’s channels 1 and 2, respectively.

- Make a Bode plot of the magnitude and phase of \( v_{out} / v_{in} \) as a function of the angular frequency. Make sure to get data over several decades of frequency values. To measure the phase, use the same technique used in Experiment-1.
- Determine experimentally the frequency at which the 3dB breaking point occurs. Compare your experimental results with the expected theoretical value.
- Determine experimentally the change of the output signal in dB per octave and per decade.

![Fig. 7 Low-pass RC filter.](image)
III.1B Two-stage low-pass filter.
Implement two RC Low-Pass filters in cascade. Repeat the same measurement requested in part III.1A.

![Diagram of two low-pass RC filters](image.png)

**Fig. 8** Two low-pass RC filters.

- Make a Bode plot of the magnitude and phase of $v_{\text{out}}/v_{\text{in}}$ as a function of the angular frequency. Make sure to get data over several decades of frequency values. To measure the phase, use the same technique used in Experiment-1.
- Determine experimentally the frequency at which the 3dB breaking point occurs. Compare your experimental results with the expected theoretical value.
- Determine experimentally the change of the output signal in dB per octave and per decade.
Calculation of $V_2$

1. \[ V_i \rightarrow \frac{V_i}{T} \rightarrow \frac{V_i}{R + 2c} = V_i \] \[ V_1 = \frac{V_i}{T} \]

2. \[ V_2 = \frac{V_i}{R + 2c} \]

But $V_1' \neq \frac{V_i}{R + 2c}$

that is

\[ V_2 \neq \frac{V_i}{R + 2c} \]

3. \[ V_2 \neq \frac{V_i}{R + 2c} \]

Calculation of $V_1'$

4. \[ i = \frac{V_i}{R + \left( \frac{1}{2c} + \frac{1}{R + 2c} \right)^{-1}} \]

5. \[ i = i_1 + i_2 \]

\[ \frac{V_i}{R + \left( \frac{1}{2c} + \frac{1}{R + 2c} \right)^{-1}} = \frac{V_i}{2c} + \frac{V_i}{R + 2c} = \left( \frac{1}{2c} + \frac{1}{R + 2c} \right) V_1' \]

\[ \frac{V_i}{R \left( \frac{1}{2c} + \frac{1}{R + 2c} \right) + 1} = V_1' \]
\[ V_1' = V_i \frac{Z_c}{R+Z_c} \frac{1}{\frac{Z_c}{R+Z_c} \left[ 1 + R \left( \frac{1}{Z_c} + \frac{1}{R+Z_c} \right) \right]} \]

\[ \frac{1}{\frac{Z_c}{R+Z_c} \left[ 2 + R + \frac{R^2 c}{R+Z_c} \right]} \]

\[ 1 \]

\[ J + \frac{Z_c R}{(R+Z_c)^2} \]

\[ V_1' = V_i \frac{Z_c}{R+Z_c} \frac{1}{J + \frac{Z_c R}{(R+Z_c)^2}} \]

\[ V_2 = \frac{V_1'}{Z_c} = V_i \left( \frac{Z_c}{R+Z_c} \right)^2 \frac{1}{1 + \frac{Z_c R}{(R+Z_c)^2}} \]

Calculation of \( V_2 \)

\[ V_2 = \frac{V_i}{\left( \frac{Z_c}{R+Z_c} \right)^2 + \frac{R}{Z_c}} = V_i \frac{1}{\left( 1 + \frac{R}{Z_c} \right)^2 + \frac{R}{Z_c}} \]

\[ \text{Compare with } \textcircled{3} \]

\[ \text{Compare with } \textcircled{3} \]