Fresnel ellipsoid of an uniaxial crystal

Ellipsoid of revolution around the optical axis

\[ n_z = n_y \neq n_x \]

Let's call
\[ n_y = n_z \equiv n_0 \]
\[ n_x \equiv n_e \]

I. PROPAGATION ALONG A PRINCIPAL AXIS

I.A CASE: Light propagating in the direction of the optical axis

Light polarized in the direction indicated by the red arrow and propagating in the direction of the optical axis will travel with speed \( c/n_0 \).
Notice that in this particular case and due to the symmetry, light polarized in ANY direction perpendicular to the optical axis will travel with speed $c/n_o$. 
I.B CASE: Light propagates in a direction perpendicular to the optical axis

Assuming $n_0 < n_e$, the two cases presented above suggests that, for a wave propagating in an arbitrary direction (different than the direction along the principal axis) its speed $v$ would have a value in the range

$$c/n_e < v < c/n_o$$

It turns out, however, that for an arbitrary direction of propagation, there always exist a wave that propagates at speed $c/n_o$. This fact is demonstrated below.
II. PROPAGATION ALONG AN ARBITRARY DIRECTION

Ellipsoid of revolution around the optical axis

\[ n_y = n_z \equiv n_o \]
\[ n_x \equiv n_e \]

Notice, for an arbitrary wave propagation direction \( \mathbf{\mu} \):

1) there always exists a direction \( \mathbf{CO} \) such that the distance \( \mathbf{CO} \) is equal to \( n_o \).
2) \( \mathbf{CO} \) is perpendicular to the plane defined by the optical axis and the direction of propagation \( \mathbf{\mu} \).
In consequence,

For every direction of propagation $\hat{\mathbf{n}}$, it is always possible to find a polarization state (where the polarization is perpendicular to the plane formed by $\hat{\mathbf{n}}$ and the optical axis, like CO in the figure above), in which the corresponding wave travels in the direction of $\hat{\mathbf{n}}$ with speed $v = c/n_o$.

That is, the speed of this wave is independent of the orientation of $\hat{\mathbf{n}}$. This lightwave is called the **ORDINARY WAVE**.

The vector displacement $\mathbf{D}_o$ and the electric field $\mathbf{E}_o$ are parallel in this case (we will justify this statement in Section 5).

On the other hand, the direction $\mathbf{CF}$ in the figure above determines the orientation of the vector displacement $\mathbf{D}_e$ of another independent wave. The length length of $\mathbf{CF}$ gives $n_e(\theta)$, which determines the speed of the wave $v = c/n_e$. That is, the speed of this wave depends on the orientation of $\hat{\mathbf{n}}$.

This lightwave is called the **EXTRAORDINARY WAVE**.

The vectors $\mathbf{D}_e$ and $\mathbf{E}_e$ are not parallel in this case (as it will be demonstrated in Section 5).

**NOTATION:** FOR UNIAXIAL CRYSTALS

$n_3 = n_2 \equiv n_o \quad \eta_1 \neq \eta_e$
In uniaxial crystals \( (n_3 = n_2 = n_0, n_1 = n_e) \)

- The difference \( \Delta n = (n_e - n_0) \) is a measure of the birefringence

- Calcite \( \Delta n < 0 \) ("negative uniaxial")
  \( \text{CaCO}_3 \)
  \( n_0 = 1.6584 \)
  \( n_e = 1.4864 \)

- Quartz \( \Delta n > 0 \) ("positive uniaxial")
  (crystallized \( \text{SiO}_2 \))

- Hexagonal, tetragonal and trigonal crystalline systems
$n_y = n_z \equiv n_o$

$n_x \equiv n_e$

Ellipsoid of revolution around the optical axis

Electric field of the ordinary wave

$E_o$

$D_o$, $E_o$, and $u$ are in the same plane

$D_e$, $E_e$, and $u$ are in the same plane (we will justify this in Section 5)