Propagation of electromagnetic waves in an anisotropic medium

\[ k_2 > k_1 \]

External force \( F \) will cause a displacement of the mass \( m \)

Notice:

\[ \vec{d} \]

The displacement \( \vec{d} \) may not be necessarily parallel to the force \( \vec{F} \) (unless \( k_2 = k_1 \))

Similarly, an incident linearly polarized electric field \( \vec{E} \) may cause electrons (inside a crystal) to oscillate along a direction \( \vec{p} \) that is not parallel to \( \vec{E} \).
This situation is similar to the rotational motion of a rigid body, where the angular momentum $\mathbf{L}$ in general is not parallel to the angular velocity $\mathbf{\omega}$, unless the rotation takes place around a principal axis of the body.

Similarly, there are at least three perpendicular directions called "principal axes" along which $\mathbf{E}$ and $\mathbf{P}$ turn out to be parallel.

Beams of linearly polarized light traveling with $\mathbf{E}$ parallel to the principal axes will have in general, different speeds.
That is, the index of refraction the beams experience while traveling through the crystal, depend on the orientation of the polarizations.

Accordingly, three principal indexes of refraction $\eta_1$, $\eta_2$ and $\eta_3$ are defined, each one associated with the 3 principal axis of the crystal.

This was the approach adopted by A. Fresnel who described the propagation of light in anisotropic material in geometrical methods long before the theory of electromagnetic came into existence.
Segment $\overline{KL} = n_1$,
$\overline{CL} = n_2$
$\overline{CM} = n_3$

Consider an electromagnetic wave traveling in the direction $\mathbf{u}$.

Intersection of the Fresnel's ellipsoid with a plane $\mathbf{u}$ to $\mathbf{u}$ and passing through $C$.
\( \overrightarrow{C_P} \) and \( \overrightarrow{C_Q} \) (mutually orthogonal) are two particular directions in terms of which we can describe the wave that propagates in the direction \( \overrightarrow{\epsilon} \).

Length of segment \( \overrightarrow{C_P} = n_p \)

The electric field, with polarization parallel to \( \overrightarrow{C_P} \), travels with speed \( \frac{c}{n_p} \).

Length of segment \( \overrightarrow{C_Q} = n_a \)

The electric field polarized along the direction \( \overrightarrow{C_Q} \) will travel with speed \( \frac{c}{n_a} \).

Note: The points \( P \) and \( Q \), along the ellipse of intersection, where chosen arbitrarily. Any others point (along the ellipse of intersection) can be chosen.
\[ \eta_x = \eta_y = \eta_z \]

Do not present special direction of polarization (Fresnel's ellipsoid reduces to a sphere)

\[ \eta_x \neq \eta_y = \eta_z \]

Show birefringent properties

The direction corresponding to the unequal index of refraction \( \eta_x \) is called optical axis.