Huygen's Principle of Wave Propagation

Wavefront: the leading surface of a wave disturbance

Huygens' Principle:

Every point on a propagating wavefront \( \Sigma \) serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelop of these wavelets.
For Huygens, only the tangent points of the wavelets to the envelope count.

The remainder of each wavelet is to be disregarded (otherwise he could not derive rectilinear propagation of light).
Huygens
- Disregarded the overlapping of the wavelets into the region of geometric shadow
- Ignored the wavefront formed by the back half of the wavelets

Despite weaknesses in his model, Huygen was able to apply his principle to prove the laws of reflection and refraction

\[ \theta_{\text{incident}} = \theta_{\text{reflected}} \]

\[ n_i \sin \theta_{\text{incid}} = n_r \sin \theta_{\text{refracted}} \]
REFLECTION at an interface

As the incident wave reaches an atom, it emits a spherical wavelet.

The envelope of the spherical wavelets constitute the wavefront of the reflected wave.
But we know light doesn't cast perfect shadows. It goes around edges or spreads out after passing narrow apertures. So, Fresnel added to the Huygens' Principle the condition that the wavelets are allowed to interfere at any point.

"Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelet (with the same frequency as that of the primary wave)

The amplitude of the optical field at any point beyond is the superposition of all these wavelets, considering their amplitudes and relative phases."

\[ K \Delta = \frac{2\pi}{\lambda} \]
All points of a wavefront serve as point sources of spherical secondary wavelets. After a time $t$, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

Draw a surface tangent to these wavelets to obtain the wavefront at the time $t$. 

**Construction of wavefronts (Isotropic materials)**

The figures present three cases for different the index of refraction of medium-2. The index of refraction is the same for medium-1. The incidence angle is the same in all the three cases. Notice the variation of the refracted angle.
REFRACTION OF LIGHT

Case: $V_2 > V1$

Glass

Air

Wavefront

Wavelet

Ray

Wavefront

$\eta_2 < \eta_1$

$V_2 > V1$

$T = \frac{1}{f} = \frac{2\pi}{\omega}$
REFRACTION OF LIGHT

Case: $V_2 < V_1$

At the boundary interface, the crests of wave 1 should coincide with the crests of wave 2.

$\theta_i$: incident angle

$\theta_r$: refraction angle
Notice:

\[ AB \sin \theta_i = \lambda_1, \]
\[ AB \sin \theta_e = \lambda_2, \]

\[ \Rightarrow \frac{\sin \theta_i}{\sin \theta_e} = \frac{\lambda_1}{\lambda_2} \]

\[ \lambda_1 = V_1 \cdot T \]
\[ \lambda_2 = V_2 \cdot T \]

\[ \Rightarrow \frac{\sin \theta_i}{\sin \theta_e} = \frac{V_1}{V_2} \]

\[ V_1 = \frac{c}{n_1}, \]
\[ V_2 = \frac{c}{n_2} \]

\[ \Rightarrow \frac{\sin \theta_i}{\sin \theta_e} = \frac{n_2}{n_1} \]

\[ n_1 \sin \theta_i = n_2 \sin \theta_e \]

Snell's Law
\[ n_1 \sin \theta_i = n_2 \sin \theta_e \]
**CASE 1**

\[ n_1 = n_2 \]

**CASE 2**

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

\[ n_1 < n_2 \Rightarrow \theta_1 > \theta_2 \]

**CASE 3**

\[ n_2 \sin(\theta_2) = n_1 \sin(\theta_1) \]

\[ n_2 > n_1 \Rightarrow \theta_2 < \theta_1 \]

What happens if we keep increasing the incidence angle \( \theta_2 \)?
Given $n_2$ and $n_1$, the critical angle $\theta_c$ can be obtained from the Snell's law

$$n_2 \sin(\theta_c) = n_1 \sin(90^\circ)$$

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

Notice this result requires $n_1 < n_2$.

What happens if we make the incidence angle greater than $\theta_c$?
"TOTAL INTERNAL REFLECTION"

$\theta_1 > \theta_c$

EVADESSENT ELECTRIC FIELD

"The further away from the interface, the weaker the field, it decreases exponentially"
Notice: there can be 2 types of incident polarized light

1. $E_{\|}$ lies in the plane of incidence

2. $E_{\perp}$ perpendicular to the plane of incidence
Reflection angle = incident angle

We have this result $\theta_i = \theta_r$ somewhat independent of the type of polarization of the incident wave.

What is the effect of the polarization characteristic of light?

When we take into account the polarization, we realize that it rather affects the intensity (power) of the reflected and transmitted light.

There exists cases in which even the reflected wave is completely suppressed! Let's see, qualitatively, how this may happen.
dipoles in the glass surface, vibrating \( \perp \) to the \( Z \)-axis.

**Notice:** There will always be a reflected wave.

dipoles in the glass surface vibrate along this direction.
$\theta_b$ is called the Brewster angle.

$\theta_0 + \alpha = 90^\circ$

$\theta_0 = \theta_b$ is such that the incident angle $\theta_0$ is reflected at $90^\circ$.

This situation happens when the wave is not reflected.

in the no reflected wave case that $\theta = 0$?
In conclusion: If $\theta_{\text{incident}} = \theta_B$ (Brewster angle)

condition to obtain polarized reflected light.

How to find $\theta_B$? If we apply Snell's law, we obtain

\[ n_1 \sin \theta_B = n_2 \sin \alpha \]

\[ = n_2 \sin (90 - \theta_B) \]

\[ = n_2 \cos \theta_B \]

\[ \Rightarrow \tan \theta_B = \frac{n_2}{n_1} \]
Application

mirror

$\theta_B$

crystal

mirror
Example:

(a) At what angle of incidence will the light reflected from water be completely polarized?

(b) Does this angle depend on the wavelength of the light?

Solution:

(a) From Table 34-1 (p. 818)

Water (20°C) \( \eta = 1.33 \)

\[
\tan(\theta_b) = \frac{\eta_{\text{water}}}{\eta_{\text{air}}}
\]

(b) ?