Huygens' Principle of Wave Propagation

Wavefront: the leading surface of a wave disturbance

The wavefront $\Sigma$ will be distorted after passing the piece of glass. How to determine the new wavefront $\Sigma'$?

Huygens' Principle:
Every point on a propagating wavefront $\Sigma$ serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelop of these wavelets.
For Huygens, only the tangent points of the wavelets to the envelop count.

The remainder of each wavelet is to be disregarded (otherwise he could not derive rectilinear propagation.)
Huygens

- Disregarded the overlapping of the wavelets in the region of geometric shadow.

- Ignored the wavefront formed by the back half of the wavelets

Despite weaknesses in his model, Huygens was able to apply his principle to prove the laws of reflection and refraction (as shown in the next sections.)

\[
\theta_{\text{incident}} = \theta_{\text{reflected}}
\]

\[
\eta_\text{s} \cdot \sin \theta_{\text{incid}} = \eta_\text{r} \cdot \sin \theta_{\text{reflected}}
\]
As the incident wave reaches an atom on the interface, the latter emits a spherical wavelet.

The envelop of the spherical wavelets constitutes the wavefront of the REFLECTED wave.
HUYGENS - FRESNEL PRINCIPLE

But we know light does not cast perfect shadows. Instead, it goes around edges or spreads out after passing narrow apertures. So, Fresnel added to the Huygens' Principle the condition that the wavelets are allowed to interfere at any point.

"Every unobstructed point of a wavefront, at a given instant, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave.) The amplitude of the optical field at any point beyond is the superposition of all these wavelets, considering their amplitude and relative phases"
All points of a wavefront serve as point sources of spherical secondary wavelets.

After a time $t$, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

Draw a surface tangent to the wavelets in order to obtain the wavefront at the time $t$. 
Construction of wavefronts (Isotropic materials)

The figures present three cases for different the index of refraction of medium-2. The index of refraction is the same for medium-1. The incidence angle is the same in all the three cases. Notice the variation of the refracted angle.

Criterion used in the construction of the wavefront

\[ t_{BC} = t_{AD} \]
REFRACTION OF LIGHT

Case: $n_2 < n_1$
$v_2 > v_1$

T: Temporal period of the wave
$T = \frac{1}{f} = \frac{2\pi}{\omega}$

1. GLASS
   - Incident wave of frequency $\omega$
   - Ray
   - Wavefront
   - Wave crests
   - $t_{BC} = T$

2. AIR
   - Wavelet
   - $t_{AD} = T$
   - Ray
   - Wavefront

$n_2$
$n_1$
REFRACTION OF LIGHT

Case: \( n_1 < n_2 \)

\( v_1 > v_2 \)

At the boundary interface, the crest of the incident wave should concide with the crests of the refracted wave.
Notice:

\[ AC \sin \theta_1 = \lambda_1 \]
\[ AC \sin \theta_2 = \lambda_2 \]

\[ \Rightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} \]

\[ \lambda_1 = v_1 T \]
\[ \lambda_2 = v_2 T \]

\[ \Rightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} \]

\[ v_1 = \frac{c}{n_1} \]
\[ v_2 = \frac{c}{n_2} \]

\[ \Rightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} \quad \text{Snell's Law} \]

That is, Huygen's principle leads to Snell's law.

This result gives confidence in using the Huygen's principle to describe the propagation of a wavefront through materials of different indices of refraction.
**CASE 1**

\[ n_1 = n_2 \]

**CASE 2**

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

\[ n_1 < n_2 \Rightarrow \theta_1 > \theta_2 \]

**CASE 3**

\[ n_2 \sin(\theta_2) = n_1 \sin(\theta_1) \]

\[ n_2 > n_1 \Rightarrow \theta_2 < \theta_1 \]

What happens if we keep increasing the incidence angle \( \theta_2 \)?
Given $n_2$ and $n_1$, the critical angle $\theta_c$ can be obtained from the Snell's Law:

$$n_2 \sin(\theta_c) = n_1 \sin(90^\circ)$$

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

Notice this result requires $n_1 < n_2$

What happens if we make the incidence angle greater than $\theta_c$?
"Total internal reflection"

$\theta_1 > \theta_c$

Evanescent electric field:
The further away from the interface, the weaker the field.
It decreases exponentially.

Waveguide:
Air → Glass → Air → Optical fiber
Notice: there can be 2 types of incident polarized light

1. $E_r$ lies in the plane of incidence

2. $E_\phi$ perpendicular to the plane of incidence