11.4 Image formation and the resolving power of a lens

Condition for distinguishing two points that are very close to each other

- P focuses at T
  The condition for having a clear image (T) of the point object P is that different rays take an equal time to travel from the source point (P) to the image point (T).

- A point P' will focus at a region very close to T.
  If P' is too close to P, their images will superimpose. The image would become too blur to distinguish the individual points P and P'.

Let's call \( t(PST) \) the traveling time that light takes to go from P to T passing through S.

The condition that ensures point P' focuses at a different point than T is,

\[
t(P'ST) \neq t(PST)
\]  
(1)
(Because if \( t(P'ST) \) were equal to \( t(PST) \), the principle of reversibility would imply that these two paths are equivalent. That is, if these two time were equal, it would mean that \( P \) and \( P' \) focus at the same point \( T \).)

But, how much different do \( t(P'ST) \) and \( t(PST) \) have to be so that we can conclude that these two paths do not belong to a set of rays forming a clear image at \( T \)?

- **The condition of resolution**
  Starting with the path PST and its corresponding time \( t(PST) \), let’s calculate the time \( t(P'ST) \) as the point \( P' \) moves out of axis away from \( P \).
  The condition of resolution (the ability to distinguish \( P \) and \( P' \)) states that

  “Two different point sources (\( P \) and \( P' \)) can be resolved only if one source (\( P \)) is imaged at such a point (\( T \)) that the time for the light rays from the other source (\( P' \)) to reach that point (\( T \)) differs by more than one period.”

In other words,

\[
  t(P'ST) - t(PST) > \text{one period of oscillation (of the radiation being used for imaging)} \\
  t(P'ST) - t(PST) > \frac{1}{\nu} \quad (\text{inverse of frequency})
\]

Using the figure below, and in terms of \( d \) and \( \theta \), the expression above becomes,

\[
  [ t(P'ST) - t(PST) ] = d \sin \theta / \nu = d \sin \theta / (c/n),
\]

where \( n \) is the index of refraction of the medium.
Thus, according to (2), the condition of resolution can be expressed as,

\[ d \sin \theta / (c/n) > \frac{1}{v} \]

Or,

\[ d > \frac{\lambda}{n \sin \theta} \]  \hspace{1cm} (3)

Thus, \( \lambda / n \sin \theta \) is the minimum distance that two points P and P’ need to be separated in order to be imaged as two different points. The latter then quantifies the resolving power of a lens:

\[ \delta = \frac{\lambda}{n \sin \theta} \]  \hspace{1cm} (4)

The quantity in the denominator is defined as the numeral aperture of the lens,

\[ NA \equiv n \sin \theta \]  \hspace{1cm} (5)

and the resolving power is typically expressed as

\[ Resolving \ power \ R = \frac{\lambda}{NA} \]  \hspace{1cm} (6)
Notice, the higher the numerical aperture of the lens, the finer the resolution.

1 Feynman Lectures, Vol-I, Page 27-8