Teleportation of an atomic state between two cavities using nonlocal microwave fields

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Implementing the ideas of Bennett et al. [Phys. Rev. Lett. 70, 1895 (1993)], we present an experimentally feasible scheme for the teleportation of an unknown atomic state between two high-Q cavities containing a nonlocal quantum superposition of microwave field states. This experiment provides alternative tests of quantum nonlocality involving high-order atomic correlations.

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Quantum nonlocality is one of the most striking predictions of modern physics [2,3]. Two quantum-correlated systems cannot be considered to be independent even if they are far apart. Local hidden variable theories lead to results concerning correlation measurements in contradiction with the quantum-mechanical predictions [3] verified in several experiments [4].

Possible implications of quantum nonlocality have ranged from cryptography [5] to computers [6]. More recently, Bennett et al. [1] have shown that an entangled pair of spin-\(\frac{1}{2}\) particles could be used, with the addition of information transmitted through a classical channel, to teleport an unknown quantum state from one observer to another. Teleportation, according to their scheme, would involve measurements made by one of the observers on four possible independent entangled states, consisting of the particle to be replicated and one of the spins of the correlated pair. Information on these measurements, transmitted through classical channels to the other observer, would allow him to reconstruct the original state on the second spin of the correlated pair, even though the original state remains necessarily unknown to the first observer, since he disposes of only one particle.

Cavity quantum electrodynamics provides new methods to build and measure nonclassical coherent superpositions of states of the electromagnetic field [8]. It is in particular possible to prepare nonlocal field states simultaneously occupying two cavities [9,10]. We show here that such nonlocal fields can be used to build a "teleportation machine": an atom, sent across the first cavity, has its quantum state replicated on another atom sent across the second cavity. As opposed to previous discussions of this question, we describe in a concrete way the sequence of measurements necessary to teleport the state. We also estimate the efficiency of detection necessary for teleporting a state with a certain precision. Our discussion makes clear the fact that the proposed scheme constitutes a new generation of high-order atomic correlation experiments. Bell's inequalities, as well as the experiments discussed in [9], refer to measurements on correlated pairs of particles. Teleportation involves on the other hand at least three-atom correlations (the scheme proposed in this paper involves actually a four-atom correlation).

A sketch of the teleportation experiment is displayed in Fig. 1. The setup consists of two identical and initially empty high-Q cavities (\(C_1\) and \(C_2\)), and three atomic beams (\(C\), \(A\), and \(B\)) made of identical two-level atoms (levels \(|e\rangle\) and \(|g\rangle\)). The \(e\rightarrow g\) transition is close to resonance with the cavity mode frequency. After switching on beam \(C\), the first atom \(c\) of this beam that crosses the two cavities establishes the nonlocal correlation between them. The atom \(a\) to be duplicated belongs to beam \(A\), which crosses only \(C_1\). Its state is reconstructed on an atom \(b\) of beam \(B\), which crosses only \(C_2\). In practice, \(e\) and \(g\) must be circular Rydberg levels with adjacent principal quantum numbers. Due to their
strong coupling to microwaves and their very long radiative decay times, circular levels are ideally suited \[11\] for preparing and detecting long-lived correlations between atom and field states. Atoms in circular Rydberg states, which can be prepared on each beam at a given time with a well-defined velocity, are counted with high efficiency by state-selective field ionization detectors \(D_a, D_b,\) and \(D_c\). By applying timed sequences of pulsed electric fields on the cavity mirrors, and taking advantage of the Stark effect of \(e\) and \(g\), the atoms can be tuned in and out of resonance, making the atom-cavity interactions resonant or dispersive during a preset time interval. Auxiliary microwave zones \((R_1, R_2\) on beam \(A, R_3\) on beam \(B)\) play the role of atomic state “polarizers” and “analyzers” and are used to perform the manipulations required by the teleportation scheme. Finally, two other microwave zones \(P_a\) on beam \(A\) and \(P_b\) on beam \(B\) are employed to prepare the state to be teleported and to analyze the fidelity of the teleportation process.

The teleportation machine is first prepared by sending across both cavities an atom \(c\) in state \(|e⟩\). This atom is made resonant with the cavities and undergoes, on the \(e\rightarrow g\) transition, a \(\pi/2\) pulse in \(C_1\) and a \(\pi\) pulse in \(C_2\). This can be easily achieved by proper setting, through Stark field adjustments, the times during which the atom is resonant with each cavity. The atomic transitions are accompanied by corresponding photon number changes. When \(c\) has undergone the first \(\pi/2\) pulse, the second cavity is still empty and the “atom \(c + C_1\)” system is in a state which corresponds to a linear superposition with equal weights of the \(e\) and \(g\) atomic states correlated to zero and one photon, respectively, in \(C_1\). If \(c\) is still in level \(e\) after leaving \(C_1\) in its vacuum state, it will, with unit probability, be transferred to \(g\) by the \(\pi\) pulse in \(C_2\) and leave a photon in the second cavity. If it emits a photon in \(C_1\) and exits it in level \(g\), it will be unaffected by its coupling with the vacuum in \(C_2\) and will leave the second cavity empty. It is thus seen that the atom always exits \(C_2\) in state \(g\), while the field is left in the entangled state

\[
|\Psi⟩=(|0⟩_1|1⟩_2+|1⟩_1|0⟩_2)/\sqrt{2},
\]

where the index 1 or 2 refers to the first or second cavity, respectively. The presence of one photon \((|1⟩)\) in either one of the cavities implies that the other is in the vacuum state \((|0⟩)\), with a maximal quantum entanglement between the two possibilities. Once atom \(c\) is detected in \(D_c\), the “teleportation machine” is ready and one can send across \(C_1\) the atom \(a\) to be teleported.

This atom is prepared by the microwave zone \(P_a\) in an arbitrary \(e, g\) superposition \(|\phi_a⟩=c_e|e⟩_a+c_g|g⟩_a\) (supposedly unknown to the observers). The combined “atom \(a + \text{field}\)” state is then the tensor product of \(|\Psi⟩\) and \(|\phi_a⟩\). This product can be conveniently expanded as

\[
|\Psi⟩=\frac{1}{2}(|\Psi^+)⟩(c_e|1⟩_2+c_g|0⟩_2)+|\Psi^-)⟩(c_e|1⟩_2-c_g|0⟩_2)
+|\Phi^+)⟩(c_e|0⟩_2+c_g|1⟩_2)+|\Phi^-)⟩(c_e|0⟩_2-c_g|1⟩_2),
\]

where we have introduced Bell’s basis \[13\] of the “atom \(a + C_1\)” states:

\[
|\Psi^±⟩=\frac{1}{\sqrt{2}}(|e⟩_a|0⟩_1±|g⟩_a|1⟩_1),
\]

\[
|\Phi^±⟩=\frac{1}{\sqrt{2}}(|e⟩_a|1⟩_1±|g⟩_a|0⟩_1).
\]

Each Bell state of “\(a + C_1\)” is correlated with a certain superposition of one- and zero-photon field states in \(C_2\), which contains information on the state of the atom to be teleported. Bennett et al.’s idea \[1\] transposed to our situation is to perform a measurement on “\(a + C_1\)” which collapses this system in one of the Bell states, automatically projecting the \(C_2\) field in one of the four combinations appearing in Eq. (2). These combinations are obtained from the initial state \(|\phi_a⟩\) by known unitary transformations of a two-level system in which \(|0⟩_2\) and \(|1⟩_2\) have replaced \(|e⟩_a\) and \(|g⟩_a\). Our teleportation problem is thus twofold: (i) How to perform on “\(a + C_1\)” a measurement whose eigenstates are given by Eq. (2)? This important point was not addressed in \[1\] and only partially solved in \[12\]. (ii) How to replicate on an atom \(b\) the information contained in the \(C_2\) field state?

Let us start with the first question, which requires a two-step approach and involves appropriate atomic manipulations in zones \(R_1\) and \(R_2\). These zones are first set so that \(a\) undergoes \(\pi/2\) pulses in each of them. Furthermore, \(a\) is tuned to have a dispersive interaction with the field in \(C_1\). This setup is equivalent to a recently demonstrated Ramsey atomic interferometer \[11\]. For a given setting of the microwave in \(R_1\) and \(R_2\), the probability for an atom to undergo an \(e\rightarrow g\) transition exhibits fringes versus the photon number in \(C_1\). Changing this photon number does indeed shift the atomic transition frequency (light shift effect), which translates into a periodic change of the \(e\rightarrow g\) transition probability induced in the two separated oscillatory field zones \(R_1\) and \(R_2\). By choosing properly the detuning between \(a\) and \(C_1\), the phase of the fringes can be shifted by \(\pi\) when the photon number varies by one unit. Moreover, the Ramsey interferometer can be adjusted so that the \(e\rightarrow g\) transfer probability is one when \(C_1\) is empty (and thus zero when \(C_1\) contains one photon). Since the atom-field interaction is dispersive, the photon number in the cavity always remains unchanged. When \(a\) crosses the interferometer, the “\(a + C_1\)” system thus...
undergoes the transformations:  
\[ |e_a\rangle|0\rangle_1 \rightarrow -|g_a\rangle|0\rangle_1, \]
\[ |e'_a\rangle|1\rangle_1 \rightarrow -|e_a\rangle|1\rangle_1, \]
\[ |g_a\rangle|0\rangle_1 \rightarrow -|e_a\rangle|0\rangle_1, \]
\[ |g_a\rangle|1\rangle_1 \rightarrow -|g_a\rangle|1\rangle_1, \]
which can be derived from the formulas given in Ref. [8]. Applying these transformations to the Bell states of Eq. (3a), one gets
\[
|\Psi^{(+)\rangle} \rightarrow -|g_a\rangle \frac{1}{\sqrt{2}} (|0\rangle_1 \mp |1\rangle_1), \tag{4a}
\]
\[
|\Phi^{(+)\rangle} \rightarrow -|e_a\rangle \frac{1}{\sqrt{2}} (|1\rangle_1 \mp |0\rangle_1). \tag{4b}
\]

State selective detection of atom \(a\) by \(D_a\) thus indicates whether the \(a + C_1\) system is in a “\(\Psi\)” or a “\(\Phi\)” Bell state.

In order to completely determine the \(a + C_1\) state, one must now decide between the alternatives left for each possible outcome of the measurement on atom \(a\). This can be done by sending through the same system a second reference atom \(a'\), prepared in the \(|g\rangle\) state. Fields in \(P_a\) and \(R_1\) are now switched off and \(a'\) is tuned to interact resonantly with \(C_1\), undergoing a \(\pi\) pulse and leaving the cavity empty if it initially contains one photon. The second zone \(R_2\) still produces a \(\pi/2\) pulse. The joint state of the system atom \(a' + C_1\) thus evolves in the following way:

\[
|g_{a'}\rangle \frac{1}{\sqrt{2}}(|0\rangle_1 \pm |1\rangle_1) \rightarrow \frac{1}{\sqrt{2}}(|e_{a'}\rangle \pm |e_{a'}\rangle) |0\rangle_1 \rightarrow \frac{1}{2} (|e_{a'}\rangle |0\rangle_1, \tag{5}
\]

The first arrow in this equation refers to the transformation undergone when \(a'\) crosses \(C_1\) and the second to the evolution in \(R_2\).

Therefore, after measuring atoms \(a\) and \(a'\), one gets complete information on the Bell state characterizing the \(a + C_1\) system, with the following correspondences:

\[ g_a, g_{a'} \rightarrow |\Psi^{(+)\rangle}, \]
\[ g_a, e_{a'} \rightarrow |\Psi^{(-)\rangle}, \]
\[ e_a, g_{a'} \rightarrow |\Phi^{(+)\rangle}, \]
\[ e_a, e_{a'} \rightarrow |\Phi^{(-)\rangle}. \]

We have then fulfilled the requirement (i), and, after the measurement on the \(a, a'\) atom pair, \(C_2\) contains a field that, within a known unitary transformation, replicates the unknown state \(\phi_a\). If we want to replicate this state on an atom, we must now address question (ii). Atom \(b\), prepared in state \(|g\rangle\), is then sent across \(C_2\) and tuned to resonance in order to produce a \(\pi\) pulse if there is one photon in \(C_2\). In this case, it leaves the cavity in the vacuum state: \(|g_b\rangle|1\rangle_2 \rightarrow |e_{b}\rangle |0\rangle_2\). On the other hand, nothing happens to the system if \(C_2\) is in the vacuum state before atom \(b\) crosses it. In this way, the information stored in the field state is completely transferred to atom \(b\):

\[ (\alpha |1\rangle_2 + \beta |0\rangle_2) |g_b\rangle \rightarrow (\alpha |e_b\rangle + \beta |g_b\rangle) |0\rangle_2, \tag{6}
\]

with \(\alpha, \beta = \pm e_e, \pm e_g\). Atom \(b\) thus leaves \(C_2\) in a state which differs from \(\phi_a\) by a known unitary transformation. Applying the inverse transformation in \(R_3\), one can thus reconstruct the initial state, completing the teleportation scheme. Note that the setting of the \(R_3\) microwave zone requires the knowledge of the \(a, a'\) measurement outcomes, which has to be transferred from \(D_a\) to \(R_3\) by a classical information channel (“wire” in Fig. 1). The teleportation scheme is thus completed, in a realistic way, on atoms crossing the two cavities separately.

We have assumed so far perfect atomic detection efficiency, which is certainly not achieved in a real experiment. Inefficient detection will affect the average success of the teleportation scheme. In order to measure the fidelity of the teleportation process, a stream of atoms \(a\) could be prepared in a well defined state by \(P_a\), and the state of a beam of atoms \(b\) could then be analyzed, with the help of the microwave zone \(P_b\) followed by the detector \(D_b\). Each measurement would imply the sampling of a large number of detection events (which would involve preparing repetitively the correlated two-cavity system and the measurements of atoms \(a, a'\), and \(b\) on a large ensemble of particles, with at least two different settings of zone \(P_b\)). If the detection efficiency is not unity, the result of these measurements will yield a two-by-two density matrix \(\rho_b\) which describes the state of atom \(b\), statistically averaged over all kinds of partially inefficient atomic detections. One can then compare the replica with the initial state by defining a teleportation fidelity coefficient for a given state \(\phi_b\) as the matrix element \(I = \langle \phi_b | \rho_b | \phi_b \rangle\). One can also define an “average” fidelity \(\hat{I}\) by averaging \(I\) over all possible states \(\phi_b\).

For perfect teleportation, we should have \(I = 1\). Easy estimates of the possible values of this quantity in several situations are obtained by representing \(|\phi_b\rangle\) in terms of spherical coordinates, which amounts to writing quite generally an atomic state as \(|\phi\rangle = \cos (\theta/2) |e\rangle + \sin (\theta/2) e^{i\phi} |g\rangle\), and

\[
I = \cos^2 \frac{\theta}{2} \rho_{b,ee} + \sin^2 \frac{\theta}{2} \rho_{b,eg} + \frac{1}{2} \sin \theta_a (e^{i\phi} \rho_{b,eg} + e^{-i\phi} \rho_{b,ee}). \tag{7}
\]

Suppose we reconstruct the state of particle \(b\) at random, without having received any information from the first cavity. This corresponds to \(\rho_b\) equal to half the unit matrix and \(I = 1/2\). Suppose now that the detection efficiency is unity, but that only the first bit of information is used to reconstruct the state (atom \(a'\) is not detected). In this case, only the \(\Phi/\Psi\) character of the Bell state is determined, and it is easy to see from Eqs. (2) and (6) that the probabilities of finding the atom in levels \(e\) and \(g\) are well reproduced, but the phases of the corresponding amplitudes are not. We should thus set \(\rho_{b,ee} = \cos^2(\theta/2)|ee\rangle\langle ee|\) and \(\rho_{b,eg} = \sin^2(\theta/2)|eg\rangle\langle eg|\) in Eq. (7), with a random phase for \(\rho_{b,eg}\). We get then

\[ I = \langle 1/2 \rangle (1 + \cos^2 \theta_a). \]

For \(\theta_a = 0\), this is equal to 1, as one should expect, since then no atomic coherence is initially present. On the other hand, for \(\theta_a = \pi/2\) we have \(I = 1/2\), equivalent therefore to complete absence of information (the original populations are then equal, and therefore transmitting just the atomic population information is equivalent to having a complete statistical mixture). The average fidelity coefficient \(\hat{I}\) when only population information is transferred is equal to 2/3.

Finally, let us consider the case in which there is no quantum coherence between the two cavities. This would correspond to having a classical alternative for the location of the single photon: if it is not found in one of the cavities, one can
then say that it was in the other cavity even before the measurement was made. This situation may be mimicked by introducing a random phase $\varphi$ in (1), which becomes then $|\Psi_2\rangle = (|0\rangle_1|1\rangle_2 + e^{i\varphi}|1\rangle_1|0\rangle_2)/\sqrt{2}$. It is easy to see that this phase, carried over to (2), is equivalent to transferring no information at all about the relative phase of $c_e$ and $c_g$, thus yielding the value $2/3$ for $\tilde{I}$. This means that, under this condition, the second bit of information (that is, the measurement of atom $a'$) becomes superfluous.

The above discussion makes it clear that $\tilde{I} = 2/3$ corresponds to the futility of “classical teleportation” [7]. In order to test quantum mechanical nonlocality, one needs therefore to have $\tilde{I} > 2/3$. We examine in the following the requirements on detection efficiency imposed by this constraint.

The triggering atom $c$ must be the first one to cross the initially empty $C_1$ and $C_2$ cavities. If $D_c$ fails to detect it, the experiment is triggered by a subsequent atom and fields having more than one photon are generated, which reduces the quantum correlation between the two cavities required for teleportation. In the worst possible case, $I$ is then reduced to $1/2$. If $\eta_a$ is the probability of detecting atom $c$, this yields: $\tilde{I} \geq \eta_a \times I_{aa'b} + (1 - \eta_a) \times \frac{1}{2}$, where $I_{aa'b}$ is the average fidelity coefficient now taking into account the detection efficiencies of atoms $a$, $a'$, and $b$ and assuming that atom $c$ is detected. A similar argument can be applied to the other atoms. If we do not detect the first atom in beam $A$, this deteriorates phase information, but does not change population information, which can be retrieved by detecting another atom in the beam, since the interaction of these atoms is dispersive. If atom $a$ is detected, but atom $a'$ is not, this would have no consequence on the population information, which depends only on the first bit. Finally, if atom $b$ is not detected, complete information would be lost, since it would absorb the single photon present in $C_2$, and the following atoms would exit in a superposition of $e$ and $g$ with equal weights. Denoting by $\eta_a$ and $\eta_b$ the detection efficiencies of $D_a$ and $D_b$ respectively, one can write therefore

$$I_{aa'b} \geq \frac{1}{2}(1 - \eta_b) + \eta_b[\eta_a^2 + \frac{\eta_a}{2}(1 - \eta_a) + \frac{1}{2}(1 - \eta_a)].$$

Assuming finally $\eta_a = \eta_b = \eta = \eta$, we get

$$\tilde{I} \geq \frac{1}{2} + \eta - \frac{\eta^3}{6} = \frac{1}{3}.$$

The condition $\tilde{I} > 2/3$ is satisfied for $\eta > 0.7$, a quite mild requirement, as compared to other tests of quantum nonlocality. The dispersion in the velocities of the atomic beams can also be easily accounted for. It would imply a departure from the ideal pulse area in each of the cavities and the Ramsey zones, thus leading to wrong bits of information. It can also be assimilated therefore to an effective efficiency. Detailed calculations show that a 10% velocity dispersion would increase the lower bound in the detection efficiency by about 5%, for $\tilde{I} > 2/3$. Of course, the total time involved in the preparation of the correlated state and the subsequent detection of the atoms should be much smaller than the dissipation time of the cavities.

We have shown that coherent nonlocal superpositions of fields in cavity quantum electrodynamics can be used for teleportation of an atomic state between two cavities. Such an experiment would provide tests of quantum nonlocality. These phenomena correspond to a new generation of experiments based on atomic correlations of higher order, such as “entanglement swapping,” [14] as opposed to the second-order correlations involved in the demonstrations of Bell’s inequalities.

Note added. We just learned that a similar scheme of cavity QED teleportation is reported by T. Sleator and H. Weinfurter (unpublished).

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FIG. 1. Sketch of the two-cavity teleportation experiment.