Fundamentals and applications of optomechanically induced transparency

Hao Xiong, and Ying Wu

Citation: Applied Physics Reviews 5, 031305 (2018); doi: 10.1063/1.5027122
View online: https://doi.org/10.1063/1.5027122
View Table of Contents: http://aip.scitation.org/toc/are/5/3
Published by the American Institute of Physics

Articles you may be interested in

Guided ionization waves: The physics of repeatability
Applied Physics Reviews 5, 031102 (2018); 10.1063/1.5031445

Chip-scale atomic devices
Applied Physics Reviews 5, 031302 (2018); 10.1063/1.5026238

Plasma physics of liquids—A focused review
Applied Physics Reviews 5, 031103 (2018); 10.1063/1.5020511

Review of force fields and intermolecular potentials used in atomistic computational materials research
Applied Physics Reviews 5, 031104 (2018); 10.1063/1.5020808

A review on high throughput roll-to-roll manufacturing of chemical vapor deposition graphene
Applied Physics Reviews 5, 031105 (2018); 10.1063/1.5035295

A practical field guide to thermoelectrics: Fundamentals, synthesis, and characterization
Applied Physics Reviews 5, 021303 (2018); 10.1063/1.5021094
Fundamentals and applications of optomechanically induced transparency

Hao Xiong and Ying Wu
School of Physics, Huazhong University of Science and Technology, Wuhan 430074, People’s Republic of China

(Received 27 February 2018; accepted 16 July 2018; published online 21 August 2018)

Cavity optomechanical systems have been shown to exhibit an analogon to atomic electromagnetically induced transparency that a transmission window for the propagation of the probe field is induced by a strong control field when the resonance condition is met. Sharp transmission features controlled by the control laser beam enable many applications ranging from force sensors to quantum communication. In recent years, there has been significant progress in both theoretical and experimental studies of this phenomenon, driven by the development of nanophotonics as well as the improvement of nano-fabrication techniques. Optomechanically induced transparency has been found to manifest in numerous different physical mechanisms, e.g., nonlinear optomechanically induced transparency, double optomechanically induced transparency, parity-time symmetric optomechanically induced transparency, and optomechanically induced transparency in various hybrid optomechanical systems, etc. These results offer a pathway towards an integrated quantum optomechanical memory, show the utility of these chip-scale optomechanical systems for optical buffering, amplification, and filtering of microwave-over-optical signals, and may be applicable to modern optical networks and future quantum networks. Here, we systematically review the latest research progress on the fundamentals and applications of optomechanically induced transparency. Perspectives and opportunities on future developments are also provided by focusing on several promising topics. Published by AIP Publishing. https://doi.org/10.1063/1.5027122

TABLE OF CONTENTS

I. INTRODUCTION .................................................. 1

II. PHYSICAL MECHANISM OF OPTOMECHANICALLY INDUCED TRANSPARENCY .................................................. 3

A. Standard configuration of optomechanically induced transparency .................................................. 3
B. Steady states of optomechanical dynamics .................................................. 4
C. Theoretical model of optomechanically induced transparency .................................................. 4
D. Optomechanically induced transparency and optomechanical nonlinearity .................................................. 5
E. Double optomechanically induced transparency .................................................. 7
F. Optomechanically induced transparency in a quadratically coupled optomechanical system .................................................. 8

III. OPTOMECHANICALLY INDUCED TRANSPARENCY IN HYBRID OPTOMECHANICAL SYSTEMS .................................................. 9

A. Induced transparency in atom-assisted optomechanical system .................................................. 9
B. Hybrid optomechanical systems with nitrogen-vacancy centers .................................................. 10
C. Optomechanically induced transparency in the microwave/X-ray regime .................................................. 10
D. Optomechanically induced transparency with mechanical driving .................................................. 11
E. Optomechanically induced transparency in coupled systems .................................................. 11

IV. EXPERIMENTAL REALIZATION AND RECENT DEVELOPMENT .................................................. 12

V. APPLICATION OF OPTOMECHANICALLY INDUCED TRANSPARENCY .................................................. 16

A. Manipulation of light propagation .................................................. 16
B. Precision measurement and force sensors .................................................. 17
C. Ground state cooling of mechanical motion .................................................. 18
VI. FUTURE PERSPECTIVES AND SUMMARY .................................................. 18

I. INTRODUCTION

Light fields in a mechanically deformable microcavity can couple the optical and mechanical degree of freedom via radiation pressure, which provides an interesting bridge between nanophotonics and nanomechanics and has attracted more and more attention over the past decade due to its important applications in gravitational-wave detection, preparation of macroscopic-scale quantum entanglement, and monitoring of mechanical motion. Radiation-pressure-based optomechanical interaction can be observed in many types of setups, including whispering-gallery-mode resonators and nanobeam optomechanical crystal with localized
Equivalent optomechanical interactions have also been revealed in optical cavities with nano-particles or ultracold atoms and come in a multitude of structures with different sizes and geometries (for a detailed review, please see Sec. IV of Ref. 1).

In theory, optomechanical interaction in these structures can be well described by the movable-end-mirror model [see Fig. 1(a)] or the membrane-in-the-middle model where a partially transparent membrane is placed inside a Fabry-Perot cavity [see Fig. 1(b)]. In the former optomechanical configuration, the radiation pressure (which is proportional to the instantaneous intracavity photon number) acts directly on the mechanical degree of freedom and the optical field is linearly coupled to the mechanical displacement. In the latter configuration, the cavity detuning \( \omega(x) \) as well as the radiation pressure are periodic in the membrane displacement \( x \). If the membrane is positioned at an antinode of the intracavity standing wave, to the lowest order, the cavity detuning is shown to be \( \omega(x) \propto x^2 \) and the optical field is quadratically coupled to the mechanical displacement in this case.

Recently, experiments of cavity optomechanics have entered the resolved sideband limit where mechanical sidebands of the optical mode lie outside its linewidth. It has been shown that the intracavity optical field can modify the effective loss factor of mechanical modes and consequently leads to mechanical cooling via the optomechanical interaction when the input field is red-detuned from the cavity resonance, where photons preferentially absorb a phonon from the mechanical oscillator and scatter upwards to the cavity resonance. This situation is quite similar to laser cooling of atomic and molecular motion in a cavity.

In fact, many phenomena arising from the coherent interaction between optical fields and atoms have corresponding analogs in cavity optomechanics via mechanical effects of light. For example, optomechanical systems exhibit a mechanical analog of the Autler-Townes splitting (splitting of atomic energy levels induced by strong fields) due to the coherent interaction of the intracavity field and mechanical oscillator via radiation pressure, and it has also been demonstrated that quantum interference in the phonon excitation pathways leads to an optomechanical analog of electromagnetically induced transparency [the so-called optomechanically induced transparency (OMIT)].

Electromagnetically induced transparency is an important phenomenon that displays a conspicuous dip in the absorption spectrum of a weak probe field when multi-level atoms or molecules appropriately coupling to a strong control field. This phenomenon arises from destructive interference of electronic pathways and gives rise to a host of exotic phenomena due to the dramatic enhancement of optical nonlinearity, including ultraslow light propagation, optical information storage, optical soliton, and lasing without inversion. Electromagnetically induced transparency was first observed in cold atomic ensembles and has received much attention in the past two decades because of its unique application in nonlinear optics and information processing. Various analogs of electromagnetically induced transparency have been observed in a variety of systems (including coupled microresonators, plasmonics, and cavity optomechanics), however, with entirely different physical fundamentals and applications. Among these analogs, optomechanically induced transparency is practically implemented in a solid-state structure, which offers attractive opportunities for integrated nonlinear optics and force sensors.

Optomechanically induced transparency was first theoretically predicted in April 2010 and experimentally observed a few months later. The similar effective interaction Hamiltonian with electromagnetically induced transparency in three-level atoms and the conspicuous dip in the absorption spectrum controlled by the control laser beam confirm the quantum interference nature of the phenomenon (see Sec. II C). Both configurations of optomechanical interaction shown in Fig. 1 can support optomechanically induced transparency, however, with distinct mechanisms (see Sec. II F). Driven by the development of nanophotonics as well as the improvement of nano-fabrication techniques, optomechanically induced transparency has subsequently been found to manifest in numerous different physical mechanisms, e.g., double optomechanically induced transparency (see Sec. II E), nonlinear optomechanically induced transparency (see Sec. II D), vector optomechanically induced transparency, parity-time (PT) symmetric optomechanically induced transparency, and optomechanically induced transparency in various hybrid optomechanical systems (see Sec. III). Clearly, an increasing pattern in the article numbers (published on the topic of optomechanically induced transparency) can be seen from Fig. 2. These results offer a pathway towards an integrated quantum optomechanical memory, show the utility of these chip-scale optomechanical systems for optical buffering, amplification, and filtering of microwave-over-optical signals, and may be applicable to modern optical networks and future quantum networks. However, it still lacks a specialized review on the fundamentals and applications of optomechanically induced transparency, especially recent theoretical and experimental progresses. Here,
we systematically review the latest research progress on the fundamentals and applications of optomechanically induced transparency. Perspectives and opportunities on future developments are also provided by focusing on several promising topics.

The paper is organized as follows. In Sec. II, we describe the model of cavity optomechanics and present the Hamiltonian. Starting from the Hamiltonian, we introduce the linearization of the interaction and the basic theory of optomechanically induced transparency in the semiclassical limit. We review optomechanically induced transparency in various hybrid optomechanical systems in Sec. III. The progress of experimental realization and application of optomechanically induced transparency are reviewed in Secs. IV and V, respectively. Finally, some perspectives and opportunities on future developments of optomechanically induced transparency are discussed in Sec. VI.

II. PHYSICAL MECHANISM OF OPTOMECHANICALLY INDUCED TRANSPARENCY

A. Standard configuration of optomechanically induced transparency

The optomechanical system usually consists of a high-Q optical cavity with mechanically deformable boundary. A schematic diagram of a general optomechanical system is shown in Fig. 1(a), where the mechanically deformable boundary is treated as a movable mirror of the Fabry-Pérot cavity and consequently described by a mechanical harmonic oscillator, viz., \( b, b^\dagger \) are the annihilation (creation) operator of the cavity field, and the Hamiltonian of a mechanical harmonic oscillator is \( \hat{H}_{\text{mech}} = \hat{p}^2/2m + m\omega_m^2\hat{x}^2/2 \), where \( \hat{p}\) and \( \hat{x} \) are the momentum and position operators of the harmonic oscillator, respectively. In the alternative, we can introduce the bosonic annihilation (creation) operator \( b^\dagger (b^\dagger) \) of the mechanical harmonic oscillator, viz., \( b = (m\omega_m\hat{x} - i\hat{p})/(2\sqrt{2m\Omega_m\nu_{\text{ZPF}}}) \) and \( b^\dagger = (m\omega_m\hat{x} + i\hat{p})/(2\sqrt{2m\Omega_m\nu_{\text{ZPF}}}) \), where \( \nu_{\text{ZPF}} = \sqrt{\hbar/2m\Omega_m} \) is the zero-point fluctuation amplitude of the mechanical oscillation, then the mechanical Hamiltonian reads \( \hat{H}_{\text{mech}} = \hbar\Omega_m(b^\dagger b + 1/2) \) with the commutation relations \([\hat{b}, b^\dagger] = 1\).

The intracavity field is driven by the input lasers through the photon hopping process, which can be well described by the following Hamiltonian in the rotating wave approximation:

\[
\hat{H}_{\text{drive}} = i\hbar\sqrt{\eta\kappa} \sum_i s_i (\hat{a}^\dagger e^{-i\omega_i t} - \hat{a} e^{i\omega_i t}),
\]

where \( s_i = e^{-i\theta_i}\sqrt{P_i/\hbar\Omega_i} \) is the normalized amplitude of the i-th optical field with power \( P_i \), frequency \( \omega_i \), and initial phase \( \theta_i \). In general, the total loss rate \( \kappa \) contains an intrinsic loss rate \( \kappa_0 \) and an external loss rate \( \kappa_{\text{ex}} \), where only the external loss rate \( \kappa_{\text{ex}} \) determines the coupling between the intracavity and input fields. The dimensionless coupling parameter \( \eta = \kappa_{\text{ex}}/\kappa \) can be continuously adjusted in experiments by tuning the taper-resonator gap.\(^{16}\)

Once the optical fields inside the cavity are excited, the intracavity photon can exert radiation pressure on the cavity wall and consequently change the position of the mechanically deformable boundary. Meanwhile, due to the fact that the radiation pressure depends sensitively on the position of the cavity wall, the motion of the cavity wall also influences the behavior of the intracavity field, which induces a feedback-backaction coupling between the cavity field and the mechanical oscillator.\(^1\) Such optomechanical interaction can substantially modify the cavity field via mechanical effects. The non-relativistic Hamiltonian of the optomechanical interaction is \( \hat{H}_I = \hbar G\hat{x}\hat{a}^\dagger \hat{a} \) with \( G \) being the optomechanical coupling constant, which can be derived from the electromagnetic wave equation with a time-varying boundary condition.\(^{28}\)

Using the annihilation (creation) operator \( \hat{b} (\hat{b}^\dagger) \) of the mechanical mode, the optomechanical interaction Hamiltonian can also be written as \( \hat{H}_I = \hbar \eta_0 (\hat{b} + \hat{b}^\dagger)\hat{a}^\dagger \hat{a} \), where \( \eta_0 = G\nu_{\text{ZPF}} \) is the single photon optomechanical coupling rate.

In the configuration of optomechanically induced transparency (see Fig. 3), the optomechanical system is driven by a strong control field with amplitude \( s_1 \) and frequency \( \omega_1 \) and a weak probe field with amplitude \( s_2 \) and frequency \( \omega_2 \). The control laser is injected at the red-detuned sideband of the cavity resonance, while the probe laser field is resonance to the cavity mode. In this case, the total Hamiltonian is \( \hat{H} = \hat{H}_{\text{mech}} + \hat{H}_{\text{cav}} + \hat{H}_I + \hat{H}_{\text{drive}} + \hat{H}_{\text{control}} + \hat{H}_{\text{probe}} \), where \( \hat{H}_{\text{drive}} = \hat{H}_{\text{control}} + \hat{H}_{\text{probe}} \) with \( \hat{H}_{\text{control}} = i\hbar\sqrt{\eta\kappa}s_1(\hat{a}^\dagger e^{-i\omega_1 t} - \hat{a} e^{i\omega_1 t}) \) and \( \hat{H}_{\text{probe}} = i\hbar\sqrt{\eta\kappa}(\hat{a}^\dagger e^{i\omega_2 t} - \hat{a} e^{-i\omega_2 t}) - \hat{H}_{\text{cav}} \). Based on the Hamiltonian and introducing the noise item, the dynamics of

![FIG. 2](image-url) Number of articles on optomechanically induced transparency by the published year. The concept of optomechanically induced transparency was proposed theoretically\(^{15}\) and demonstrated experimentally\(^{16}\) in 2010, with about twenty publications on this topic identified in the next year, thirty publications in 2012, over forty in 2013, over fifty in 2014 and 2015, over sixty in 2016, and over seventy in 2017.

![FIG. 3](image-url) Standard frequency configuration of optomechanically induced transparency.

\[ \frac{\text{Number of articles published}}{\text{Year}} \]


- 0
- 10
- 20
- 30
- 40
- 50
- 60
- 70
- 80

\[ \text{Number of articles published} \]

\[ \text{Year} \]
intracavity fields and the mechanical motion can be described by the Heisenberg-Langevin equations

\[
\dot{a} = (i\omega_0 - iG\chi - \kappa/2)\dot{a} + \sqrt{\eta K} \sum_{i=1,2} S_i e^{-i\omega_i t} + \sqrt{\eta K} \hat{a}_{in},
\]

\[
m \left( \frac{d^2}{dt^2} + \Omega_m^2 + \Gamma_m \frac{d}{dt} \right) \ddot{x} = -\hbar G \dot{\hat{a}}^* + \bar{F}_{th},
\]

with \( \eta = 1 - \eta \). To achieve at these equations, the dissipations with the Markov approximation are introduced and the decay rates of the cavity [the term \(-\kappa/2 \cdot \dot{\hat{a}} \) in Eq. (2)] and the mechanical oscillators [the term \( m \Gamma_m \dot{\hat{a}} \) in Eq. (3)] are introduced classically. The quantum noise of the optical and mechanical modes are described by \( \hat{a}_{in} \) and \( \bar{F}_{th} \) with \( \langle \hat{a}_{in}(t)\hat{a}_{in}^*(t') \rangle = \delta(t-t'), \langle \hat{a}_{in}(t) \rangle = 0, \langle \bar{F}_{th}(t)\bar{F}_{th}^*(t') \rangle = \Gamma_m \int e^{-i\omega(t'-t)} \left[ \coth(\hbar\omega/2k_B T) + 1 \right] d\omega/2k_B T \) and \( \langle \bar{F}_{th}(t) \rangle = 0 \).

**B. Steady states of optomechanical dynamics**

In the single photon weak-coupling regime \( g_0 \ll \kappa \), the operators can be reduced to their expectation values, viz., \( a(t) \equiv \langle \hat{a}(t) \rangle \) and \( x(t) \equiv \langle \hat{x}(t) \rangle \), and the quantum and thermal noise terms can be eliminated safely due to \( \langle \hat{a}_{in}(t) \rangle = 0 \) and \( \langle \bar{F}_{th}(t) \rangle = 0 \). In this semiclassical limit, the averaged version of the Heisenberg-Langevin equations becomes

\[
\dot{a} = \frac{i(\Delta - G\chi) - \kappa/2}{\Delta} a + \sqrt{\eta K} (s_1 + s_2 e^{-i\Delta t}),
\]

\[
m \left( \frac{d^2}{dt^2} + \Gamma_m \frac{d}{dt} + \Omega_m^2 \right) \ddot{x} = -\hbar G a^* a,
\]

where \( \Delta = \omega_1 - \omega_0 \) and \( \Omega = \omega_2 - \omega_0 \). The evolution equations (4) and (5) are nonlinear due to the optomechanical interaction \(-\hbar G a^* a\), by which the intracavity fields and the mechanical motions influence each other. When the probe laser is absent that only the control field with amplitude \( s_1 \) is incident into the cavity, the evolution equations (4) and (5) admit a steady-state solution

\[
\tilde{a} = \sqrt{\eta K s_1} / i\Delta + \kappa/2, \quad \tilde{x} = -\hbar G |\tilde{a}|^2 / m\Omega_m^2,
\]

with \( \tilde{\Delta} = \Delta - G\chi \). The system achieves at the steady state \((\tilde{a}, \tilde{x})\) in about a few microseconds after the light entering into the cavity [as shown in Figs. 4(a) and 4(b) for the optical and mechanical dynamics, respectively], determined by the decay rates of the mechanical oscillator.

Due to the nonlinear feedback mechanism, optomechanical dynamics can exhibit a bistable behavior (one input has two possible and stable outputs) in the steady-state optical response when the control field is strong enough [optical and mechanical bistable behaviors are shown in Figs. 4(c) and 4(d), respectively]. Such optomechanical bistability attracted many attention and had become one of the focuses of current research interest in cavity optomechanics, although bistability is not a unique feature of the optomechanical dynamics. It has been shown that bistable behavior can be found in various nonlinear systems, such as magnetic materials, heat transfer, and even economic evolution.

**C. Theoretical model of optomechanically induced transparency**

To observe optomechanically induced transparency, the optomechanical bistability should be absent and the probe field should be much weaker than the control field so that it can be treated as the perturbation of the steady-state. Although exact solutions to the evolution equations (4) and (5) are still unknown, many important phenomena arising from the mechanical effect of light, including optomechanically induced transparency, absorption, amplification, and mechanical cooling, can be well described by the linearization of the evolution equations around the steady state \((\tilde{a}, \tilde{x})\).

The solution of Eqs. (4) and (5) can be written as \( a = \tilde{a} + \delta a \) and \( x = \tilde{x} + \delta x \), where \( \delta a \) and \( \delta x \) obey the following linearized equations (16) (nonlinear terms of \(-iG\delta a\delta a^* \) and \(-\hbar G\delta a^* \delta a \) are omitted):

\[
d\delta a = \frac{d}{dt} (\delta a - \frac{\kappa}{2} \delta a) - iG\tilde{a} \delta x + \sqrt{\eta K s_2 e^{-i\Delta t}},
\]

\[
m \left( \frac{d^2}{dt^2} + \Omega_m^2 + \Gamma_m \frac{d}{dt} \right) \delta x = -\hbar G (\tilde{a} \delta a^* + \tilde{a}^* \delta a).
\]
The physical picture is that the control field provides a steady-state solution \((\bar{a}, \bar{x})\) of the system, while the probe field is treated as the perturbation of the steady state. Equations (7) can be solved analytically by using the ansatz: 
\[
d\bar{\alpha} = A^+ e^{-i\Delta t} + A^- e^{i\Delta t} \quad \text{and} \quad \ddot{\bar{x}} = X e^{-i\Delta t} + X e^{i\Delta t} ,
\]
which, together with the input-output relation \(^1\) between the input and output fields \(s_{in}(t) = s_{out}(t) - \sqrt{\kappa} \bar{a}\), finally leads to the analytical expression of the transmission of the probe laser \(^{16}\)
\[
tp = 1 - \frac{1 + jf(\Omega)}{\kappa/2 - i(\Delta + \Omega) + 2\Delta f(\Omega)} \eta \kappa , \quad \text{(8)}
\]
where \(f(\Omega) = hG^2 |\bar{a}|^2 \chi(\Omega)/[\kappa/2 + i(\Delta - \Omega)] \) and \(\chi(\Omega) = 1/m(\Omega^2 - \Omega^2 - i\Gamma_m \Omega)\). Using the single photon optomechanical coupling rate \(g_0\), we have \(f(\Omega) = hG^2 |\bar{a}|^2 \chi(\Omega)/[\kappa/2 + i(\Delta - \Omega)]\), which is determined by the combined parameter \(g_0 |\bar{a}|^2\). The transmission of the probe field as a function of the probe detuning \(\Omega\) shows a transmission window on resonance \((\Omega = \Omega_m)\) with the window width \(\Gamma_m + 4g_0^2 |\bar{a}|^2 / \kappa \ll \kappa\). The width of the transparency window (or dip in absorption spectrum) can be presented as the sum of the intrinsic rate \(\Gamma_m\) and the optomechanically induced damping rate \(\Gamma_{opt} = C \Gamma_m\), where \(C\) is the optomechanical cooperativity parameter defined as \(C \equiv 4g_0^2 |\bar{a}|^2 / \kappa \Gamma_m\). Optomechanically induced transparency has been discussed theoretically by Agarwal and Huang and demonstrated experimentally by Weis et al. \(^{16}\) and Safavi-Naeini et al., \(^7\) respectively.

The physical origin of the standard optomechanically induced transparency can be understood as follows. The beat of the probe field and the control field induces a time-varying radiation-pressure force with beat frequency \(\Omega = \omega_2 - \omega_1\). If \(\Omega \approx \Omega_m\) is the beat frequency matches the mechanical resonance frequency, the mechanical resonator is driven resonantly and starts to oscillate coherently. Subsequently, the mechanical oscillation leads to the creation of optical sidebands on the cavity field, generating photons at frequencies \(\omega_1 + n\Omega_m\) with \(n\) being an integer which denotes the order of the optical sideband. For the case of strong control field and in the resolved sideband limit, the first order sideband \((n = 1)\) is dominated and has the same frequency as the probe field. The destructive interference between the first order sideband and the probe field leads to the cancellation of the intracavity field and consequently results in a transparency window in the transmission. The standard optomechanically induced transparency has been well studied in the regime \(g_0 |\bar{a}| \ll \kappa \ll \Omega\) and \(\Gamma_m \ll 4g_0^2 |\bar{a}|^2 / \kappa\), and in this regime the window width of optomechanically induced transparency can be estimated as \(\sim 4g_0^2 |\bar{a}|^2 / \kappa\). The signals at higher order sidebands arise from the nonlinear nature of the optomechanical interaction and can be calculated in a perturbation analysis (see Sec. II D).

Similar to electromagnetically induced transparency in atomic systems, the phenomenon of optomechanically induced transparency can also be well explained by a typical \(\Lambda\)-type three-level system which is composed of three states \(|0_a, 0_b\rangle\), \(|0_a, 1_b\rangle\), and \(|1_a, 0_b\rangle\) (as shown in Fig. 5), with the subscripts \(a\) and \(b\) represent the photonic and phononic states, respectively. When the resonance condition is met, the probe field couples \(|0_a, 0_b\rangle \Rightarrow |1_a, 0_b\rangle\), while the control field couples \(|0_a, 1_b\rangle \Rightarrow |1_a, 0_b\rangle\). A destructive interference of these two excitation pathways occurs when \(\Omega = \Omega_m\) which leads to optomechanically induced transparency.

**D. Optomechanically induced transparency and optomechanical nonlinearity**

Optomechanically induced transparency can be well described by the linearized optomechanical interaction with \(\Delta = -\Omega_m\), viz., the strong control field drives the optomechanical system on the red sideband. On account of the nonlinearity, optomechanical interaction generates photons at second- and higher-order sidebands (with frequencies \(\omega_1 + n\Omega_m\), where \(n\) is an integer denoting the order of the optical sideband). \(^{37}\) Such second- and higher-order sideband generation is quite similar to the second and higher harmonic generation in nonlinear optics. \(^{38–40}\) By that analogy, the amplitude of second order sideband generation can be calculated analytically in a perturbation analysis by considering that the signal of the second order sideband is seed by the first order sideband of the intracavity field. In the perturbative regime, the intracavity field at the first order sideband is assumed to rise superior to the second- and higher-order sideband generation due to the intrinsic weakness of optomechanical nonlinearity, which leads to the following ansatz containing the second order sideband: 
\[
d\ddot{\alpha} = \ddot{\alpha}_1 + \ddot{\alpha}_2 + \cdots \quad \text{and} \quad \ddot{x} = \ddot{x}_1 + \ddot{x}_2 + \cdots ,
\]
with \(\ddot{\alpha}_1 = A_1 e^{-i\Delta t} + A_1 e^{i\Delta t} + A_1 e^{2i\Delta t} + A_1 e^{3i\Delta t} + \cdots \), \(\ddot{\alpha}_2 = A_2 e^{-i\Delta t} + A_2 e^{i\Delta t} + A_2 e^{2i\Delta t} + \cdots \), \(\ddot{x}_1 = X_1 e^{-i\Delta t} + X_1 e^{i\Delta t} + \cdots \), and \(\ddot{x}_2 = X_2 e^{-i\Delta t} + X_2 e^{i\Delta t} + \cdots \). Substitution of the nonlinear ansatz into Eqs. (7) leads to two set independent algebraic equations (each set contains three equations) after ignore of the higher order sideband components: one set describes the linearized optomechanical dynamics, while the other describes second-order sideband generation.

The same algebraic approach can be extended to describe nonlinear processes in a double-probe-field-driven optomechanical system, where the optomechanical system is driven by a strong pump field and two relatively weak probe fields. It has been shown that there are signals at the

![FIG. 5. Level scheme of the standard optomechanically induced transparency.](Image 322 to 360)

The probe field couples \(|0_a, 0_b\rangle \Rightarrow |1_a, 0_b\rangle\), while the control field couples \(|0_a, 1_b\rangle \Rightarrow |1_a, 0_b\rangle\). A destructive interference of these two excitation pathways occurs when \(\Omega = \Omega_m\) which leads to optomechanically induced transparency.

---

\(\Delta = -\Omega_m\), viz., the strong control field drives the optomechanical system on the red sideband. On account of the nonlinearity, optomechanical interaction generates photons at second- and higher-order sidebands (with frequencies \(\omega_1 + n\Omega_m\), where \(n\) is an integer denoting the order of the optical sideband). \(^{37}\) Such second- and higher-order sideband generation is quite similar to the second and higher harmonic generation in nonlinear optics. \(^{38–40}\) By that analogy, the amplitude of second order sideband generation can be calculated analytically in a perturbation analysis by considering that the signal of the second order sideband is seed by the first order sideband of the intracavity field. In the perturbative regime, the intracavity field at the first order sideband is assumed to rise superior to the second- and higher-order sideband generation due to the intrinsic weakness of optomechanical nonlinearity, which leads to the following ansatz containing the second order sideband: 
\[
d\ddot{\alpha} = \ddot{\alpha}_1 + \ddot{\alpha}_2 + \cdots \quad \text{and} \quad \ddot{x} = \ddot{x}_1 + \ddot{x}_2 + \cdots ,
\]
with \(\ddot{\alpha}_1 = A_1 e^{-i\Delta t} + A_1 e^{i\Delta t} + A_1 e^{2i\Delta t} + A_1 e^{3i\Delta t} + \cdots \), \(\ddot{\alpha}_2 = A_2 e^{-i\Delta t} + A_2 e^{i\Delta t} + A_2 e^{2i\Delta t} + \cdots \), \(\ddot{x}_1 = X_1 e^{-i\Delta t} + X_1 e^{i\Delta t} + \cdots \), and \(\ddot{x}_2 = X_2 e^{-i\Delta t} + X_2 e^{i\Delta t} + \cdots \). Substitution of the nonlinear ansatz into Eqs. (7) leads to two set independent algebraic equations (each set contains three equations) after ignore of the higher order sideband components: one set describes the linearized optomechanical dynamics, while the other describes second-order sideband generation.

The same algebraic approach can be extended to describe nonlinear processes in a double-probe-field-driven optomechanical system, where the optomechanical system is driven by a strong pump field and two relatively weak probe fields. It has been shown that there are signals at the