Review of Classical Analog of Electromagnetically Induced Transparency

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Abstract

Alzar, Martinez, and Nussenzveig have proposed an experiment to demonstrate electromagnetically induced transparency by using a coupled oscillator circuit made up of nothing more than passive resistors, inductors, and capacitors. I will review this paper and check their work, including generating the transfer functions for the coupled and uncoupled examples, as well as providing a possible use for this technology in the future and other modeling possibilities using this same analog of EIT.

Introduction

Look at the light around you. It has several different properties: Brightness, Color, Direction, Polarization, Velocity, and even Momentum. We know many ways to change several of these characteristics, but some are harder to change than others. What if we could not only change them, but control them as well? It has already been shown that not only can we reduce the group velocity of light down by an extraordinary degree to 17 m/s, we can even stop light\(^1\). Once we have it stopped, what can we do with it? Can we select a portion of light to release to a detector? Electromagnetically Induced Transparency (EIT) has several applications for allowing us to control an external change in velocity for a wave pulse, highly selectable by frequency.

In their paper Classical Analog of Electromagnetically Induced Transparency, Alzar, Martinez, and Nussenzveig present a simple experiment to demonstrate the coupling effects of EIT by using a pair of coupled RLC oscillators\(^2\). In this review I will examine their methods and results by performing the experiment and calculating the power transfer using standard electrical engineering methods and Middlebrooks Extra Element Theorem.

1) Theoretical Model

EIT has three general modes of operation: Ladder, Vee, and Lambda\(^3\). Ladder is a way of activating an absorption effect in a medium, while Vee and Lambda negate an already existing absorption effect. We will be focusing on the Lambda configuration for this review.

The main point of understanding is the phenomenon of coherent population trapping\(^4\). Population trapping occurs when there is a difference of an internal Raman transition between the probe laser and the coupling laser. The coupling band is unpopulated, so the coupling laser is not absorbed. However, the laser still traps the electrons in the probe band. That is, they can't transition to the destination band. Since these electrons can't transition to the destination band the probe laser no longer has any electrons to absorb the frequency. At this point transparency has occurred.
The way CPT occurs is by using the coupling laser to split the destination into two bands, one higher than the original, and one lower than the original. The difference between the Autler Townes Doublet and EIT lies solely in the strength of the coupling laser. If the coupling laser is strong the dual destination bands are fully separated. Therefore, the frequency of interest no longer results in a transition from the internal energy state to the destination band. If the coupling laser is weak the dual destination bands have a region of overlap. However, according to quantum theory, two paths that result in the same destination must interfere. This destructive interference again denies entry into the original band. This traps the electrons and creates CPT.

This electron interaction is what creates the absorption of the probe laser by the material. Once we excite the absorbent material with the coupling laser and create this coherent population trapping phenomenon the refractive index at the probe frequency becomes the same as in the outside medium. Since the changing refractive index is what causes reflections and opacity, removing the change in the refractive index causes the material to become transparent at the frequency of interest. This is what is called electromagnetically induced transparency.

Once this main point is understood there is another point of discrimination for the effect. If the intensity of the coupling beam is weak, or less than the overall decay rate, then we only see a dip at the difference frequency, the difference being the energy of the Raman transition. This dip is due to the quantum interference of two different electron paths resulting in the same destination point. If the intensity of the coupling beam is strong (more than the overall decay rate) then we see a separation of the absorbent frequencies. This separation is known as an Autler Townes Doublet. These two frequencies are separated proportionally by the intensity of the coupling beam $\Omega$. 
Velocity of Passed Light

The velocity of the light is made up of phase velocity and group velocity. Usually, phase velocity and group velocity are more or less equal. That is to say, the dispersion in most materials is small. However, the dispersion is not zero. Dispersion is the phenomenon by which the refractive index is dependent on frequency. In an ordinary medium, where the dispersion is small, we can neglect this term and simply write the following relationship:

\[ V_g = V_p = \frac{c}{n} \]

In some cases, the dispersion is not small. In an absorbent medium the dispersion varies over the absorption band. This causes the refractive index to vary. Most materials behave this way, so that one material will be opaque over the visible range of light and yet be transparent to UV waves. In most media this change is over a large range of frequencies, so the speed of light does not vary greatly. In our case, involving a very small range of frequencies, the speed of light can vary greatly.

![Optical Frequency](image)

Figure 2: Index of Refraction: The grey line shows the paired absorption bands that occur in EIT. The blue line shows the index of refraction that occurs over that region. Note the steep slope that occurs in the middle of the figure. The center value for the index of refraction is 1.

In this case just knowing the index of refraction, n, isn’t enough. Due to the high level of dispersion we have to come up with a new index of refraction \( n_g \) which pertains to the group velocity:

\[ n_g = n + \omega \frac{dn}{d\omega} \]

\[ V_g = \frac{c}{n_g} \]
In the case of EIT we have a region where the index of refraction is changing rapidly. In this case:

$$\frac{n}{\omega} \ll \frac{dn}{d\omega}$$

$$n_g \gg n$$

$$V_g \approx \frac{c}{\omega} \star \frac{1}{\frac{dn}{d\omega}}$$

If the intensity of the coupling beam is small then the two peaks will be closer together. These close peaks will reduce the bandwidth of the light that is passed through by EIT, without lowering the total change in the index of refraction. By creating a steeper slope to the index of refraction we can achieve a slower velocity of the light that is propagating at the transparent frequency.

2) EIT-like Phenomena in Coupled RLC Circuits

Now that this basis is understood we will begin a discussion relating this to the coupled harmonic oscillator circuit. This is a basic circuit studied in the first year of an undergraduate students electrical engineering curriculum, and can be created in a lab with a few resistors, inductors, and capacitors. Combine this with a few leads, probes, an oscilloscope, and a function generator, and you have a complete laboratory experiment suitable for any undergraduate electrical engineering curriculum.

![Figure 3: Circuit Model](image)

**Power Dissipation**

The first thing we have to do is examine the basic circuit. Figure 3 shows the circuit with the switch installed. The transfer function for the circuit with the switch open is simply the power dissipated. Here we calculate that using the Laplace transformations taught in basic electrical engineering. The current can be measured by measuring the voltage across $L_2$ and integrating according to the voltage vs current equation of an inductor. Then we square the current and multiply by the resistance and we have the power dissipated by the circuit.
First we measure the amplitude (A) of the voltage across the inductor (L_2) and calculate the current:

\[ P = I^2 R \]
\[ L_2 \frac{dI}{dt} = V_{L_2} \]
\[ I = \frac{1}{TL_2} \int_0^T V_{L_2} \, dt \]
\[ V_{L_2} = Ae^{i\omega t} \]
\[ I = \frac{A}{\omega L_2} e^{i\omega t} \]
\[ \text{Re}(I) = \frac{A}{\omega L_2} \cos i\omega t \]
\[ I^2 = \frac{A^2}{\omega^2 L_2} \left( \frac{1}{2} + \frac{1}{2} \cos i\omega t \right) \]
\[ I^2 = \frac{1}{2} \left( \frac{A}{\omega L_2} \right)^2 \]

Then we calculate the effective impedance (R_{open}) of the circuit with the open switch. I will make use of standard engineering circuit notation to denote parallel impedences:

\[ R_{open} = R_2 + sL_2 + \frac{1}{sC_2} + \frac{1}{sC} = R_2 + sL_2 + \frac{1}{sC_2} \]
\[ sC_{e2} = s(C||C_2) \]
\[ R_{open} = \frac{s^2L_2C_{e2} + sR_2C_{e2} + 1}{sC_{e2}} \]

The power dissipation becomes:

\[ P_{open} = \frac{1}{2} \left( \frac{A}{\omega L_2} \right)^2 \ast \frac{s^2L_2C_{e2} + sR_2C_{e2} + 1}{sC_{e2}} \]

Secondly, we calculate the power dissipation of the circuit with the switch closed. The current calculation stays the same, so all we need is a new resistance calculation:

\[ R_{closed} = R_2 + sL_2 + \frac{1}{sC_2} + \frac{1}{sC} || (R_1 + sL_1 + \frac{1}{sC_1}) \]
\[ R_{closed} = \frac{1}{s(C + C_1)} \frac{s^2L_1C_1 + sR_1C_1 + 1}{s^2L_1C_{e1} + sR_1C_{e1} + 1} + \frac{s^2L_2C_2 + sR_2C_2 + 1}{sC_2} \]

\[ \text{Re} \] of this becomes:

\[ \frac{1}{2} \left( \frac{A}{\omega L_2} \right)^2 \ast \left( \frac{1}{s(C + C_1)} \frac{s^2L_1C_1 + sR_1C_1 + 1}{s^2L_1C_{e1} + sR_1C_{e1} + 1} + \frac{s^2L_2C_2 + sR_2C_2 + 1}{sC_2} \right) \]

there will be a dip in power dissipation at the center frequency with the switch closed. The magnitude of this dip will be significant enough to cause a block at that frequency. Even more important than the magnitude, however, is the bandwidth of that dip. Before we can answer that question we will first determine whether we’re in the AT region or the EIT region.
AT vs EIT

The bandwidth of the open system is characterized by \( Q = \frac{f_0}{\Delta f} \), where
\[
Q = \frac{1}{R_2 \sqrt{\frac{L_2}{C_{c2}}}}
\]
\[
f_0 = \frac{1}{2\pi \sqrt{L_2 C_{c2}}}
\]
\[
\Delta f = \frac{f_0}{Q}
\]

The frequency difference that we’re concerned with for the system with the closed switch is actually the difference between the rejected frequency and the passed frequency of the uncoupled system. The rejected frequency is the center frequency of the uncoupled system, and the passed frequency is the frequency not involving the coupling capacitor:
\[
f_{low} = \frac{1}{2\pi \sqrt{L_2 C_2}}
\]
\[
f_0 - f_{low} = \frac{1}{2\pi \sqrt{L_2}} (\frac{1}{\sqrt{C_{c2}}} - \frac{1}{\sqrt{C_2}})
\]
\[
f_{high} - f_{low} = 2 * (f_0 - f_{low}) = \frac{1}{\pi \sqrt{L_2}} (\frac{1}{\sqrt{C_{c2}}} - \frac{1}{\sqrt{C_2}})
\]

Here we get into a discrimination between the region of Autler Townes Doublet and Electromagnetically Induced Transparency. EIT here is characterized by a small dip in the middle of the frequency spectrum and the entire spectrum of the coupled circuit fitting within the spectrum of the uncoupled circuit:
\[
2 * (f_{high} - f_{low}) < \Delta f
\]
\[
4 * (\sqrt{\frac{C_2}{C_{c2}}} - 1) < R_2 \sqrt{\frac{C_{c2}}{L_2}}
\]

After performing some algebra and using the quadratic formula we discover the simple result that, for our values, \( C > C_2 \) gives us a response in the EIT region and \( C < C_2 \) gives us a response in the AT region.

Now that we understand what region each situation produces, we can then use our our results to provide a frequency difference, or bandwidth, of the two absorption peaks:
\[
f_{high} - f_{low} = 2 * (f_0 - f_{low}) = \frac{1}{\pi \sqrt{L_2}} (\frac{1}{\sqrt{C_{c2}}} - \frac{1}{\sqrt{C_2}})
\]
\[
BW = \frac{1}{\pi \sqrt{L_2 C_2}} (\sqrt{1 + \frac{C_2}{C}} - 1)
\]

For EIT, where \( C > C_2 \), we can use the following approximation:
\[
BW = \frac{1}{2\pi \sqrt{L_2 C}} \sqrt{\frac{C_2}{C}}
\]
For AT, where $C < C_2$, we can use a different approximation:

$$BW = \frac{1}{\pi \sqrt{L_2 C}} \left(1 - \sqrt{\frac{C}{C_2}}\right)$$

These approximations work best when there’s at least a factor of 2 between $C$ and $C_2$ in both cases. This is easily checked by setting $C = C_2$ and finding that the two approximations are not equal to each other in that case.

**Transfer Matching**

Since this is a review of a paper we have to ask the question: Do these results match those obtained in that paper? This is a trivial question once you understand the Laplace Transform commonly used in electrical engineering:

$$s = j\omega$$

For the open switch the transfer function listed in the paper under review is the real part divided by the total magnitude$^1$:

$$R_2(\omega_S) = \frac{R_2}{R_2^2 + (\omega_S L_2 - 1/\omega_S C_2)^2}$$

Using engineering methods:

$$R_{\text{open}} = \frac{s^2 L_2 C_2 + s R_2 C_2 + 1}{s C_{c_2}}$$

$$R_{\text{open}} = R_2 + j(\omega L_2 - \frac{1}{\omega C_{c_2}})$$

We can easily see that the real and imaginary parts are the same, so these two methods are equivalent. This is slightly more difficult to show for the circuit created with the closed switch$^1$:

$$R_{\text{closed}} = \frac{\text{Re}(\omega_S)}{\text{Re}(\omega_S)^2 + \text{Im}(\omega_S)^2}$$

$$\text{Re}(\omega_S) = R_2 + \frac{R_1/(\omega_S C)^2}{R_1^2 + (\omega_S L_1 - 1/(\omega_S C_1))^2}$$

$$\text{Im}(\omega_S) = (\omega_S L_2 - 1/(\omega_S C_2)) - \frac{(1/(\omega_S C)^2)(\omega_S L_1 - 1/(\omega_S C_1))}{R_1^2 + (\omega_S L_1 - 1/(\omega_S C_1))^2}$$

Using engineering methods:

$$R_{\text{closed}} = \frac{1}{s(C + C_1)} \frac{s^2 L_1 C_1 + s R_1 C_1 + 1}{s^2 L_2 C_2 + s R_2 C_2 + 1}$$

$$R_{\text{closed}} = R_2 + j(\omega L_2 - \frac{1}{\omega C_2}) + \frac{1}{j\omega C} \frac{R_1}{R_1 + j(\omega L_1 - \frac{1}{\omega C_{c_1}})}$$

As you can see in each case the transfer functions are equivalent. The data shown in the next section will provide experimental proof both that the methods are correct and that they demonstrate the phenomenon of EIT in a simple system.
3) Data Analysis

The data was collected with the simple circuit from above. Data was collected as voltage amplitudes at varying frequencies. Power dissipation was then calculated using the preceding equations to acquire the graphs. Those graphs are in the following figure:

![Graph](image)

Figure 4: Results: from left to right, the values of the coupling capacitor C are 0.94 µF, 0.47 µF, 0.22 µF, 0.10 µF, and 0.05 µF. The blue curve represents power dissipation of the uncoupled oscillator circuit. The red curve represents power dissipation of the coupled oscillator circuit. The last two graphs are good examples of AT, where \( C > C_2 \), while the first two are good examples of EIT, where \( C < C_2 \). The middle graph, where \( C = C_2 \), is a good example of the critical point between the two effects. This is because the peaks of the coupled system are just within the overall envelope of the uncoupled system.

To achieve these results in the classical model I chose a frequency region that would readily allow me to perform the measurements without issue. 1 to 100 kHz is a frequency range that provides flexibility in allowing me to use inexpensive components as well as allowing me to use a standard function generator as my input. This equipment is readily available in any standard electrical engineering laboratory. The component values are listed below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>0 Ω</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>100 Ω</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>390 µH</td>
</tr>
<tr>
<td>( L_2 )</td>
<td>390 µH</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>0.22 µF</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0.22 µF</td>
</tr>
</tbody>
</table>

One point I should note. I did not use a resistor for \( R_1 \). This sets the Q of the coupling circuit to infinity which gives it a very high and very sharp profile. This accentuates the cutout at the center frequency. This is a realistic analog to the optical model. In the optical model all the dissipation effects are from the electrons in the atom trying to return to a lower energy state. These are modelled in the uncoupled oscillator. There are no additional sources of dissipation from the coupling laser, so setting \( R_1 \) to 0 Ωs is appropriate.
Conclusion

In summary, coupled electronic oscillators can model EIT and AT with some success. It must be noted that one of the main differences between the classical model and the optical model is that in the optical model the coupling frequency is at a constant offset from the probe frequency, with that offset being the Raman transition energy internal to the material. The intensity of the coupling laser determines whether we’re in the EIT or the AT region, while the ratio of the coupling frequency to the incident frequency determines the region being modelled. Furthermore, I have not been able to determine a good analog for the speed of light passing through the transparent region. I believe the magnitude of the slope of the power dissipation around the center frequency in the coupled circuit is a good analog for the magnitude of the slope of the refractive index in that region. From that we can see that as the slope becomes steeper, the speed of light in that region will slow down.

References


