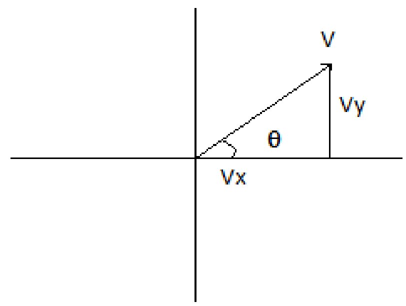
Vectors

Properties of vectors

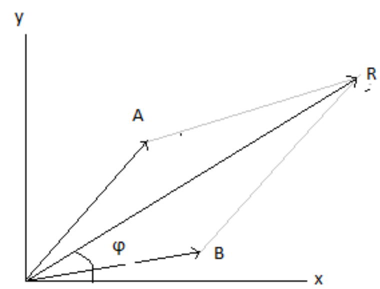
Given $\vec{A} = \langle A_x, A_y, A_z \rangle$ and $\vec{B} = \langle B_x, B_y, B_z \rangle$,

- $c\vec{A} = \langle cA_x, cA_y, cA_z \rangle$
- $\vec{A} + \vec{B} = \langle A_x + B_x,\ A_y + B_y,\ A_z + B_z \rangle$



$$V_x = \|\vec{V}\| \cos(\theta) \quad V_y = \|\vec{V}\| \sin(\theta)$$

Addition of Vectors



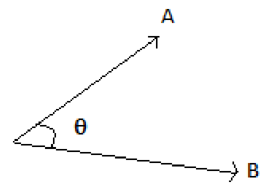
$$R_x = A_x + B_x \quad R_y = A_y + B_y$$

$$\|R\| = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\varphi = tan^{-1}\left({R_y}/{R_x}\right)$$

Vectors

Dot Product



$\vec{A} \cdot \vec{B} = \|\vec{A}\|\|\vec{B}\|\cos(\theta)$ or, given $\vec{A} = \langle A_x, A_y, A_z \rangle$ and $\vec{B} = \langle B_x, B_y, B_z \rangle$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{A} = \|A\|^2$$

*note: The dot product picks out the components of two vectors which are *parallel* to one another and leaves out the pieces which are perpendicular

Cross Product

$\|\vec{A} \times \vec{B}\| = \|A\|\|B\|\sin(\theta)$ where the right hand rule may be used to determine the orientation. Or, more formally,

$$\vec{A} \times \vec{B} = det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$=\langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

$$\vec{A} \times \vec{A} = 0$$

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

*note: the cross product picks out the pieces of each vector which are perpendicular to one another and discards the parallel components. The resultant vector is perpendicular to each of the base vectors.

Identities

- $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B})$
- $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$