Classical Mechanics

Kinematics:

Linear Motion:

Uniform acceleration:

$v = v_0 + at \quad s = s_0 + v_0 t + \frac{1}{2}at^2 \quad v^2 = {v_0}^2 + 2a\Delta s \quad \Delta s = \frac{1}{2}(v_0 + v) \quad \Delta s = vt - \frac{1}{2}at^2$

General:

$v = \frac{ds}{dt} \quad \bar{v} = \frac{\Delta s}{\Delta t} \quad a = \frac{dv}{dt} \quad \bar{a} = \frac{\Delta v}{\Delta t} \quad \Delta s = \int_{t_0}^{t_f} v dt \quad \Delta v = \int_{t_0}^{t_f} a dt$

Circular Motion:

Uniform angular acceleration:

$\omega = \omega_0 + \alpha t \quad \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \omega^2 = {\omega_0}^2 + 2\alpha\Delta\theta \quad \Delta\theta = \frac{1}{2}(\omega_0 + \omega) \quad \Delta\theta = \omega t - \frac{1}{2}\alpha t^2$

$\theta = {}^{s}/_{r} \quad \omega = \frac{v_{tan}}{r} \quad \alpha = {}^{a_{tan}}/_{r}$

General

$\omega = \frac{d\theta}{dt} \quad \bar{\omega} = \frac{\Delta\theta}{\Delta t} \quad \alpha = \frac{d\omega}{dt} \quad \bar{\alpha} = \frac{\Delta\omega}{\Delta t} \quad \Delta\theta = \int_{t_0}^{t_f} \omega dt \quad \Delta\omega = \int_{t_0}^{t_f} \alpha dt$

Centripetal:

$a_{\hat{r}} = a_c = \frac{{v_{tan}}^2}{r} = \frac{4\pi^2 r}{T^2} = \omega^2 r$

Forces:

Newton's Laws:

- An object in motion will remain at constant velocity until an external force acts upon it
- $\sum F_{ex} = ma$
- For every force, there is an equal and opposite force ($F_{ab} = -F_{ba}$)

Weight: $F_w = mg$ Friction: $F_{fr} = F_N\mu$ Wind resistance: $F_D = \frac{1}{2}CA\rho v^2$ Spring: $F_s = -kx$

Gravitational: $F_G = -\frac{Gm_1m_2}{r^2}\hat{r}$ Centripetal: $F_C = -\frac{m{v_{tan}}^2}{r}\hat{r} = -m\omega^2 r\hat{r}$

Energy:

Work: $W = \int_{s_0}^{s_f} \vec{F}\cdot\overrightarrow{ds}$ (generalized) or $W = \vec{F}\cdot\overrightarrow{\Delta s}$ (if F=constant)

$E_f = E_i + W_{nc}$ (general) or $E_f = E_i = constant$ (conservative systems) where $E = K + U$

*A force is conservative if $\nabla \times \vec{F} = 0$

Potential Energy:

Gravitational: $U_G = mgh$ (near Earth's surface) or, $U_G = \frac{Gm_1m_2}{r}$ (general) Spring: $U_s = \frac{1}{2}kx^2$

$*F = -\nabla U = -\frac{\partial U}{\partial x}\hat{\imath} - \frac{\partial U}{\partial y}\hat{\jmath} - \frac{\partial U}{\partial z}\hat{k}$

Kinetic Energy:

$K_{lin} = \frac{1}{2}mv^2 \quad K_{rot} = \frac{1}{2}I\omega^2 \quad K_{total} = K_{lin} + K_{rot}$

Power:

$P = \frac{dW}{dt}$ or, $\bar{P} = \frac{\Delta W}{\Delta t}$ $P = \vec{F}\cdot\vec{v}$ (constant force) $P = \vec{\tau}\cdot\vec{\omega}$ (constant torque)

Momentum:

Linear Momentum:

$\vec{p} = m\vec{v}$

$\sum F_{ext} = \frac{d\vec{p}}{dt}\ (general) or\ \frac{\Delta p}{\Delta t}\ (constant\ force)$

*if $\sum F_{ext} = 0$ then $\vec{p}$ is constant, or $\overrightarrow{p_f} = \overrightarrow{p_i}$

$\Delta\vec{p} = \vec{J} : \vec{J} = \int_{t_0}^{t_f} F dt\ (general)\ or\ \vec{J} = F\Delta t\ (constant\ force)$

Angular Momentum:

$L = I\omega = \vec{r}\times\vec{p}$

$\sum \tau_{ext} = \frac{dL}{dt}\ (general)\ or\ \frac{\Delta L}{\Delta t}\ (constant\ force)$

*if the sum of the external torques is zero, then angular momentum is conserved

$\Delta L = \int_{t_0}^{t_f} \tau dt\ (general)\ or\ \tau\Delta t\ (constant\ torque)$

Units for Classical Mechanics

Unit	Abbreviation	quantity	fundamental units
kilogram	kg	mass	basic
second	s	time	basic
meter	m	length	basic
Newton	N	force	kg*m/s^2
Joule	J	energy	N*m
Watt	W	power	J/s

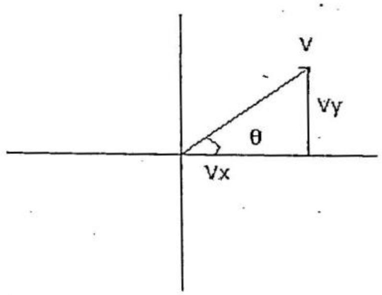
Classical Mechanics

Vectors

Properties of vectors

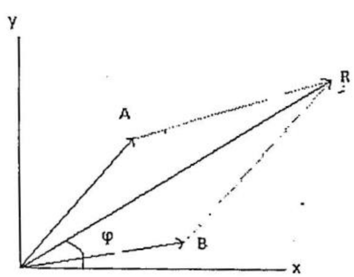
Given $\vec{A} = \langle A_x, A_y, A_z \rangle$ and $\vec{B} = \langle B_x, B_y, B_z \rangle$,

- $c\vec{A} = \langle cA_x, cA_y, cA_z \rangle$
- $\vec{A} + \vec{B} = \langle A_x + B_x,\ A_y + B_y,\ A_z + B_z \rangle$



$$V_x = \|\vec{V}\| \cos(\theta) \quad V_y = \|\vec{V}\| \sin(\theta)$$

Addition of Vectors

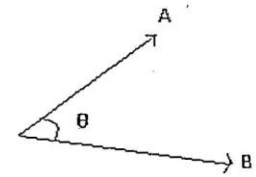


$$R_x = A_x + B_x \quad R_y = A_y + B_y$$

$$\|R\| = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\varphi = tan^{-1}\left({R_y}/{R_x} \right)$$

Dot Product



$\vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos(\theta)$ or, given $\vec{A} = \langle A_x, A_y, A_z \rangle$ and $\vec{B} = \langle B_x, B_y, B_z \rangle$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{A} = \|A\|^2$$

*note: The dot product picks out the components of two vectors which are *parallel* to one another and leaves out the pieces which are perpendicular

Cross Product

$\|\vec{A} \times \vec{B}\| = \|A\| \|B\| \sin(\theta)$ where the right hand rule may be used to determine the orientation.

Or, more formally,

$$\vec{A} \times \vec{B} = det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \langle A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x \rangle$$

$$\vec{A} \times \vec{A} = 0$$

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

*note: the cross product picks out the pieces of each vector which are perpendicular to one another and discards the parallel components. The resultant vector is perpendicular to each of the base vectors.

Identities

- $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B})$
- $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$