STATISTICS 241 FORMULAS

Common Symbols

n = Sample Size

$\bar{x}$ = Sample Mean

s = Sample Standard Deviation

s^2 = Sample Variance

$\hat{p}$ = Sample Proportion

r = Sample Correlation Coefficient

N = Population Size

μ = Population Mean

σ = Population Standard Deviation

σ^2 = Population Variance

p = Population Proportion

ρ = Population Correlation Coefficient

Descriptive Statistics

In Excel use Data > Data Analysis > Descriptive Statisitcs

$$\bar{x} = \frac{\Sigma x}{n} \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} \qquad s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} \qquad z = \frac{x-\bar{x}}{s}$$

Range = Max – Min

IQR=interquartile range = $Q_3 - Q_1$

Coefficient of Variation = $\frac{s}{\bar{x}}(100\%)$

Finding Outlier Limits: Lower = $Q_1 - (1.5*IQR)$ Upper = $Q_3 + (1.5*IQR)$

Probability

Union Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Complement Rule: $P(A^C) = 1 - P(A)$

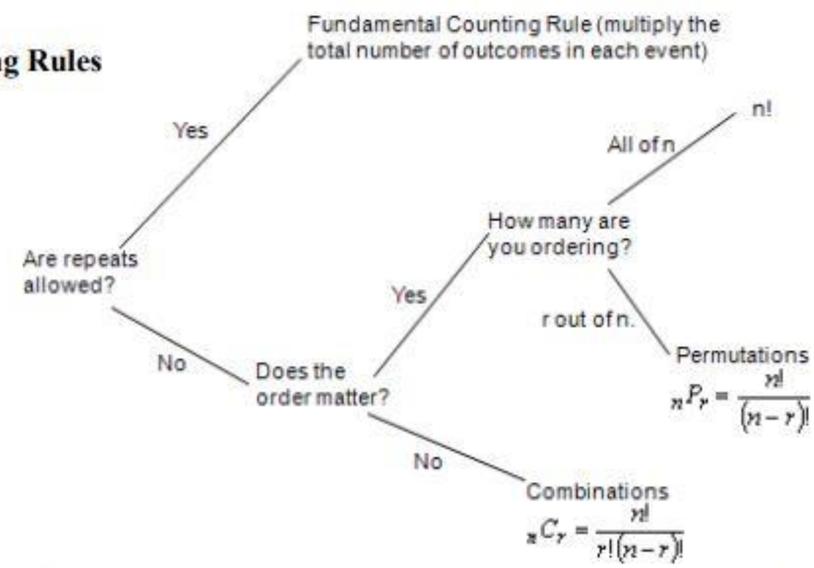
Conditional Probability: $P(A\,|\,B) = \frac{P(A \cap B)}{P(B)}$

Mutually Exclusive Events: $P(A \cap B) = 0$

Dependent Events: $P(A \cap B) = P(B) \cdot P(A\,|\,B)$

Independent Events: $P(A \cap B) = P(B) \cdot P(A)$

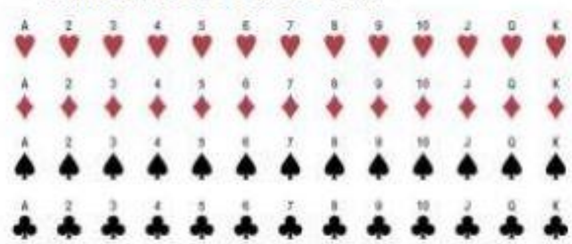
Counting Rules



Sum of Two Dice

		Second Die					
	+	1	2	3	4	5	6
First Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Standard Deck of Cards



Face Cards are Jack (J), Queen (Q) and King (K)

Source: Rachel L. Webb, PSU, January 2018

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Discrete Probability Distributions

$E(x) = \mu = \Sigma x \cdot P(x) \qquad \sigma^2 = \left(\Sigma x^2 \cdot P(x)\right) - \mu^2 \qquad \sigma = \sqrt{\left(\Sigma x^2 \cdot P(x)\right) - \mu^2}$

Binomial Distribution: $P(x) = {}_nC_x \cdot p^x \cdot q^{(n-x)}$ $\qquad \mu = n \cdot p \qquad \sigma^2 = n \cdot p \cdot q \qquad \sigma = \sqrt{n \cdot p \cdot q}$

Signs Are Important For Discrete Distributions!		
P(X=x)	**P(X≤x)**	**P(X≥x)**
Is	Is less than or equal to	Is greater than or equal to
Is equal to	Is at most	Is at least
Is exactly the same as	Is not greater than	Is not less than
Has not changed from	Within	
Is the same as		
=BINOM.DIST(x,n,p,false)	=BINOM.DIST(x,n,p,true)	=1-BINOM.DIST(x−1,n,p,true)
TI: binompdf(n,p,x)	TI: binomcdf(n,p,x)	TI: 1−binomcdf(n,p,x−1)
Where:	**P(X>x)**	**P(X<x)**
x is the value in the	More than	Less than
question that you are	Greater than	Below
finding the probability for.	Above	Lower than
p is the proportion of a	Higher than	Shorter than
success expressed as a	Longer than	Smaller than
decimal between 0 and 1	Bigger than	Decreased
n is the sample size	Increased	Reduced
	Smaller	Larger
Excel:	=1−BINOM.DIST(x,n,p,true)	=BINOM.DIST(x−1,n,p,true)
TI-Calculator:	1−binomcdf(n,p,x)	binomcdf(n,p,x−1)

Poisson Distribution: $P(x) = \frac{e^{-\mu}\mu^x}{x!}$ Change mean to fit the units in the question: $\text{new } \mu = \text{old } \mu \times \left(\frac{\text{new units}}{\text{old units}}\right)$

P(X=x)	**P(X≤x)**	**P(X≥x)**
Is	Is less than or equal to	Is greater than or equal to
Is equal to	Is at most	Is at least
Is exactly the same as	Is not greater than	Is not less than
Has not changed from	Within	
Is the same as		
=POISSON.DIST(x,mean,false)	=POISSON.DIST(x,mean,true)	=1−POISSON.DIST(x−1,mean,true)
TI: poissonpdf(mean,x)	TI: poissoncdf(mean,x)	TI: 1−poissoncdf(mean,x−1)
Where:	**P(X>x)**	**P(X<x)**
x is the value in the	More than	Less than
question that you are	Greater than	Below
finding the probability for.	Above	Lower than
	Higher than	Shorter than
The mean has been rescaled to	Longer than	Smaller than
the units of the question.	Bigger than	Decreased
	Increased	Reduced
	Smaller	Larger
Excel:	=1−POISSON.DIST(x,mean,true)	=POISSON.DIST(x−1,mean,true)
TI-Calculator:	1−poissoncdf(mean,x)	poissoncdf(mean,x−1)

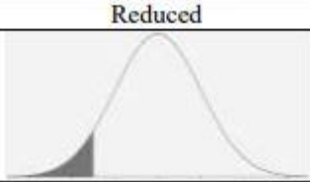
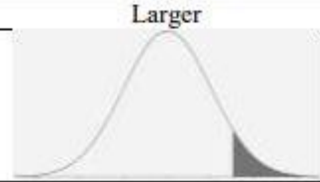
STATISTICS 241 FORMULAS

Continuous Distributions

$z = \frac{x-\mu}{\sigma}$ $\quad$ $x = z \cdot \sigma + \mu$ $\quad$ Central Limit Theorem: $z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

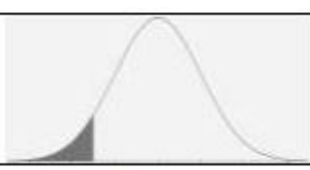
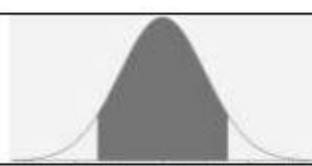
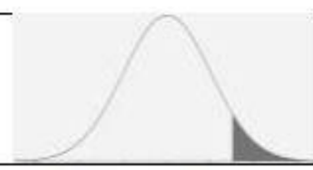
Note that for a continuous distribution there is no area at a line under the curve, so ≥ and > will have the same probability and use the same Excel commands.

Normal Distribution

Normal Distribution Finding a Probability		
$P(X \le x)$ or $P(X<x)$	$P(x_1<X<x_2)$ or $P(x_1 \le X \le x_2)$	$P(X \ge x)$ or $P(X>x)$
Is less than or equal to	Between	Is greater than or equal to
Is at most		Is at least
Is not greater than		Is not less than
Within		More than
Less than		Greater than
Below		Above
Lower than		Higher than
Shorter than		Longer than
Smaller than		Bigger than
Decreased		Increased
Reduced		Larger
		
=NORM.DIST (x,μ ,σ,true)	=NORM.DIST (x_2,μ ,σ,true)- NORM.DIST (x_1,μ ,σ,true)	=1- NORM.DIST (x,μ ,σ,true)

Note that the NORM.S.DIST function is for a standard normal when μ=0 and σ=1.

Inverse Normal Distribution

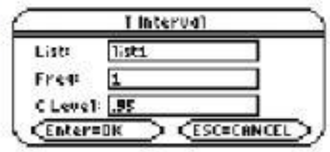
Normal Distribution Finding an X-value Given an Area or Probability		
$P(X \le x)$ or $P(X<x)$	$P(x_1<X<x_2)$ or $P(x_1 \le X \le x_2)$	$P(X \ge x)$ or $P(X>x)$
Lower	Between	Upper
Bottom		Top
Below		Above
Reduced		More than
Less than		Greater than
Lower than		Larger
Shorter than		Higher than
Smaller than		Longer than
Decreased		Bigger than
		Increased
		
=NORM.INV(area,μ,σ)	x_1 =NORM.INV(1-area/2,μ ,σ) x_2 = | x_1|	=NORM.INV(1-area, μ,σ)

STATISTICS 241 FORMULAS

Confidence Intervals **When n < 30 the variable must be approximately normally distributed.**

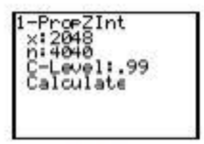
t-Interval **The 100(1 - α)% confidence interval for μ, σ is unknown, is** $\bar{x} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$ **.**

- On the TI-83 you can find a confidence interval using the statistics menu. Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the [8:TInterval] option and press the [ENTER] key. Arrow over to the [Stats] menu and press the [ENTER] key. Then type in the mean, sample standard deviation, sample size and confidence level, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the answer in interval notation. Be careful, if you accidentally use the [7:ZInterval] option you would get the wrong answer.
- *Or* (If you have raw data in list one) Arrow over to the [Data] menu and press the [ENTER] key. Then type in the list name, L_1, leave Freq:1 alone, enter the confidence level, arrow down to [Calculate] and press the [ENTER] key.
- On the TI-89 go to the [Apps] **Stat/List Editor**, then select 2nd then F7 [Ints], then select **1:TInterval**. Choose the input method, data is when you have entered **data** into a list previously or **stats** when you are given the mean and standard deviation already. Type in the mean, standard deviation, sample size (or list name (list1), and Freq: 1) and confidence level, and press the [ENTER] key. The calculator returns the answer in interval notation. Be careful, if you accidentally use the [1:ZInterval] option you would get the wrong answer.

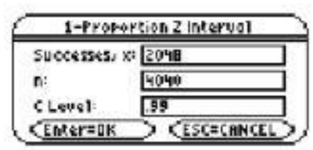


1 Proportion z-Interval $\hat{p} \pm z_{\alpha/2}\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}$

- On the TI-84 press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the [A:1-PropZInterval] option and press the [ENTER] key. Then type in the values for X, sample size and confidence level, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the answer in interval notation. Note: sometimes you are not given the x value but a percentage instead. To find the x to use in the calculator, multiply $\hat{p}$ by the sample size and round off to the nearest integer. The calculator will give you an error message if you put in a decimal for x or n. For example if $\hat{p} = .22$ and n = 124 then .22*124 = 27.28, so use x = 27.



- On the TI-89 go to the [Apps] **Stat/List Editor**, then select 2nd then F7 [Ints], then select **5: 1-PropZInt**. Type in the values for X, sample size and confidence level, and press the [ENTER] key. The calculator returns the answer in interval notation. Note: sometimes you are not given the x value but a percentage instead. To find the x value to use in the calculator, multiply $\overline{p}$ by the sample size and round off to the nearest integer. The calculator will give you an error message if you put in a decimal for x or n. For example if $\overline{p} = .22$ and n = 124 then .22*124 = 27.28, so use x = 27.



Sample Size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$ $n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{E}\right)^2$ Always round n up to the next integer.

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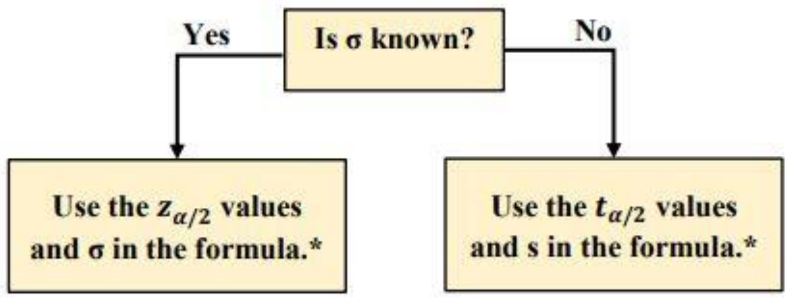
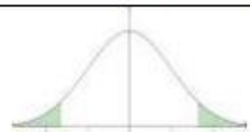
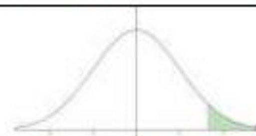
Hypothesis Testing

α = P(Type I error)



***If n < 30, the variable must be normally**

Look for these key words to help set up your hypotheses:

Two-tailed Test	Right-tailed Test	Left-tailed Test
$H_0 : \mu = \mu_0 \quad H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0 \quad H_1 : \mu > \mu_0$	$H_0 : \mu = \mu_0 \quad H_1 : \mu < \mu_0$
		
Claim is in the Null Hypothesis		
=	≤	≥
Is equal to	Is less than or equal to	Is greater than or equal to
Is exactly the same as	Is at most	Is at least
Has not changed from	Is not more than	Is not less than
Is the same as	Within	
Claim is in the Alternative Hypothesis		
≠	>	<
Is not	More than	Less than
Is not equal to	Greater than	Below
Is different from	Above	Lower than
Has changed from	Higher than	Shorter than
Is not the same as	Longer than	Smaller than
	Bigger than	Decreased
	Increased	Reduced

The rejection rule:

- p-value method: reject H_0 when the p-value ≤ α.
- Critical value method: reject H_0 when the test statistic is in the critical tail(s).
- Confidence Interval method, reject H_0 when the hypothesized value found in H_0 is outside the bounds of the confidence interval.

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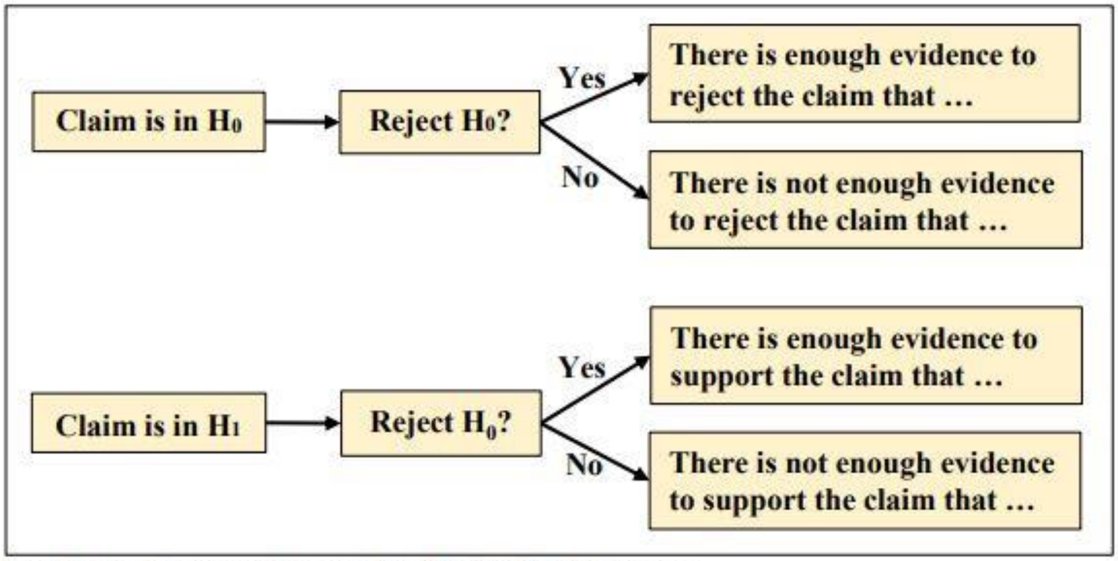

Finish conclusion with context and units from question.

One Sample Tests:

1-Sample Mean t-test: $H_0: \mu = \mu_0$, $H_1: \mu \neq \mu_0$ Test statistic when σ is unknown: $t = \dfrac{\bar{x} - \mu_0}{(s/\sqrt{n})}$ with df= n – 1

1-Sample Proportion z-test: $H_0: p = p_0$, $H_1: p \neq p_0$ Test statistic is $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$ where $\hat{p} = \dfrac{x}{n}$

Two Sample Tests:

2-Sample Means t-test – Independent Populations

$H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$ or $H_0: \mu_1 - \mu_2 = (\mu_1 - \mu_2)_0$, $H_1: \mu_1 - \mu_2 \neq (\mu_1 - \mu_2)_0$ where usually you have $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \neq 0$

Note that $(\mu_1 - \mu_2)_0 = 0$ in most cases

Assuming Equal Variances

Test statistic when σ_1^2 and σ_2^2 are unknown: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left(\left(\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}\right)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)\right)}}$

Assuming Unequal Variances

Test statistic when σ_1^2 and σ_2^2 are unknown: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ with

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\left(\left(\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2\right) + \left(\dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2\right)\right)}$$

2-Sample Means t-test – Dependent Populations

Find the difference (d) between each matched pairs. $H_0: \mu_D = 0$, $H_1: \mu_D \neq 0$ Test statistic: $t = \dfrac{\bar{d} - \mu_0}{(s_d/\sqrt{n})}$

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2 Proportions

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

Test statistic $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$, (usually $(p_1 - p_2)_0 = 0$) where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$, $\hat{q} = 1 - \hat{p}$

Correlation and Regression

$H_0: \rho = 0$

$H_1: \rho \neq 0$

Test statistic for correlation: $t = r\sqrt{\left(\dfrac{(n-2)}{(1-r^2)}\right)}$ with df= n – 2 $\quad \hat{y} = a + bx$

r = sample correlation coefficient $\quad \rho$ = population correlation coefficient

a = y-intercept $\quad$ b = slope $\quad$ s = s_{est} = standard error of estimate $\quad R^2$ = coefficient of determination

One-Factor ANOVA table k=#of groups, N=total of all n's

H_0: $\mu_1=\mu_2=\mu_3=\ldots=\mu_k$

H_1: At least one mean is different $\quad$ CV: Always a right-tailed F, use Excel =F.INV.RT(α, df_B, df_W)

ANOVA Table

Source	SS	df	MS	F (Test Statistic)
Between (Treatment or Factor)	$SS_B=\Sigma n(\bar{x}-\bar{x}_{GM})^2$	k–1	$MS_B = SS_B / df_B$	$F = MS_B / MS_W$
Within (Error)	$SS_W=\Sigma(n-1)s^2$	N–k	$MS_W = SS_W / df_W$	
Total	SS_T	N–1		

When you reject H_0 for a one-factor ANOVA then you should do a multiple comparison. For example for 3 groups you would have the following 3 comparisons. (4 groups would have ${}_4C_2$=6 comparisons)

$H_0:\mu_1=\mu_2 \quad H_0:\mu_1=\mu_3 \quad H_0:\mu_2=\mu_3$

$H_1:\mu_1\neq\mu_2 \quad H_1:\mu_1\neq\mu_3 \quad H_1:\mu_2\neq\mu_3$

- Bonferroni Test $t = \dfrac{(\bar{x}_i - \bar{x}_j)}{\sqrt{\left(MSE\left(\dfrac{1}{n_i} + \dfrac{1}{n_j}\right)\right)}}$ with df = N – k and to get the p-value you would multiply the tail areas by ${}_kC_2$ groups. For example if you use the tcdf in your calculator to find the area in both the tails and you have 4 groups you would multiply the tail areas by ${}_4C_2$=6.

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