Using Sommerfeld's approach, the self-demagnetizing field along the centerline of the magnetic coating on a recording tape was calculated for an isolated transition from

$$\phi = - \int \frac{\Delta I}{r} dV - \int \frac{I_n}{r} dS, \quad (1)$$

and

$$H = - \nabla \phi, \quad (2)$$

where $\phi$ is the magnetic scalar potential, and $H$ is the self-demagnetizing field for points inside the tape coating. Because only the longitudinal component of the magnetization for an isolated transition was under consideration, the surface integral in (1) was negligible. Also, because the approximation used for the magnetization distribution was independent of depth and lateral position in the tape, the magnetization divergence $\Delta I$ in the volume integral was identical with $d(I_n \arctan \frac{a}{x})/dx$.

Miyata and Hartel performed the volume integration of the expression resulting from the above simplifications, and calculated the resulting field at any point. This author pointed out that the sign associated with their result should be a minus sign rather than their plus sign. With the sign change, and the $2M_x/r$ value for $I_n$, the longitudinal field (a self-demagnetizing field) along the recording track for which the centerline of the tape coating is

$$H_{\text{self-demag.}} = -4M_x \left[ \arctan \left( \frac{a}{2} \right) \right]_x$$

$$= \arctan \left( \frac{a}{2} \right)$$

$$H_{\text{self-demag.}} = 0, \quad \text{when } x = 0. \quad (3)$$

By setting $x$ in (3) equal to the value which maximizes $H_{\text{self-demag.}}$, and then setting (3) equal to the medium's coercivity $H_c$, it was possible to plot the ratio $H_c/4M_x$ ($= H_c/B_x$) as a function of the scale factor $a$, where $a$ was in units of the medium's half-thickness $b/2$, and where $B_x$ was the medium's retentivity. These values of $a$ are then the values which correspond to the shortest possible magnetization transitions for different ratios of $H_c$ to $B_x$.

Because the read head scans the slope of the recorded magnetization, the distance along the recording track for which the $d(2M_x/r \arctan x/a)/dx$ exceeded 20 per cent of its maximum amplitude was then defined as the length of the transition zone. The 20 per cent level was chosen because a number of magnetic recording systems use this value as the clipping level. The reciprocals of the transition lengths for the above values of $a$ were then plotted as a function of the ratio $H_c/B_x$. This graph is shown in Fig. 1, and describes the approximate theoretical limit on NRZI bit density for saturation-type recording when the track width is large compared to the thickness of the medium. The specific values indicated for red oxide coatings of 0.3 mil and 0.5 mil thicknesses are in reasonable agreement with the reciprocal of Teer's experimental pulse widths for Gevaert and BASF L.G.R. tapes, respectively.

**Gain of Multisampler Systems**

In a recent correspondence [8], G. C. Reis gave another "simplest" method for determining the output of a multisampler system. Of course, which of the several available methods [1]–[8] is the simplest is open to individual judgment; however, I do have a few comments concerning the one proposed by Reis.

Firstly, his basic premise that Mason's technique can be applied directly to the original block diagram representation of the sampled system (with the modifications he describes) requires proof. Fortunately, this proof is contained in the paper by Lendaris and Jury [1]. Even so, caution has to be exercised in doing this, because Mason's technique gives a gain between two nodes, which may be called the input and output nodes. For an arbitrary sampled-data system, a transfer gain from the input to the output nodes per se does not exist because the input itself cannot always be separated from the other terms (of the Z transform) to form this desired ratio. Reis did not indicate how to cope with this problem in his note.

Reis argues that his method is simpler because it consists of only one step, writing the answer. This is misleading because there is indeed an intermediate step (albeit a mental one), namely, determining whether

or not either of his conditions 1) or 2) with their attendant restrictions apply, and if so to follow the steps he gives to write down the answer. If the restrictions of the two conditions are violated, he promises that the method provides a "graph reduction process for arriving at a solution," whatever this means.

I wish to again point out that the method given by Lendaris [1], [3], [6] can be applied directly to multisampler systems (linear and with synchronous samplers) of any configuration, with no worry about satisfying certain constraints, etc.; and that this method consists of making a very straightforward transformation (with a trivial amount of "thinking") from the given block diagram to a special kind of signal-flow graph to which Mason's theorem can be directly applied with no "ifs" or "buts." The method for effecting the transformation is given [1], [3], [5] as a 6-step procedure and, really, seems much longer in writing than it is in practice.

**Author's Comment**

I wish to thank Dr. Lendaris for his comments and hope that the following will clarify some of the points raised.

Firstly, it is agreed that the "simplest" solution to a problem is a function of both the problem and the problem-solver.

Secondly, it should be pointed out that Mason's signal-flow graph technique solves the following type of problem: Given the signal at one node of the graph, what is the signal at some other node? This is the same type of problem which is of interest in the sampled-data case. However, as Lendaris points out, a "gain" in the sense of output-over-input cannot be found, in general. Perhaps the word "output" should be substituted for "gain" in the title of my letter, since the method proposed determines the output as some function of the input (as do all present methods).

Thirdly, by a "graph reduction process" I mean the process of reducing the number of paths of a graph in some logical manner. In the general multiloop system which I was considering, it may happen that paths imbedded in some of the loop subgraphs (or some loop subgraphs themselves) meet the two conditions of the recent correspondence. Thus the signal at some node (call it b) can be expressed as some function [say f(a)] of the signal at some other node (call it a). Thus all paths between nodes b and a can be removed from the graph and replaced by a source f(a) (which can be determined by inspection) at node b. Although this may not simplify detailed calculations, it does provide economy of thought in arriving at an expression for the output.

Lastly, I hasten to admit that the conditions required are restrictive and therefore the technique I proposed lacks the gen-
The Assistor, a Component with Bipolar Negative Resistance, Part II*  

A few months ago I had told briefly, under this title, of a solid-state device consisting of two electrodes (lead wires) with a thin layer of semiconductor between them. Much has since been done to understand and develop the device.

Testing many combinations of electrodes and semiconductors I have found that the voltage-vs-current characteristics of, perhaps, the greater part exhibited, for both polarities, voltage peaks followed by smoothly falling regions. In a variety of assemblies suited to control pressure or spacing, flat-dimensioned, wire and poled—equal or different—electrodes were tested, of aluminum, copper, gold, iron, mercury, silver, zinc, various alloys, and graphite; as semiconductors:—cupric oxide CuO, ferric oxide FeO₃, zinc oxide ZnO, cadmium sulfide CdS, lead sulfide PbS, and others; mostly as fine powder settled on one electrode from a thin suspension in a drop of water. Typical peak voltages varied then from one to ten volts, at currents near 50 to 100 μA; negative resistance often exceeded -5 kohm. All characteristics were traced on an oscilloscope with sinewaves of 1.2 kHz, often gated 50/sec to 5 k/sec, to minimize vacuum. Negative resistances were also checked by use in a 1.5 kHz sinewave oscillator and with a small-signal neg-ohm-meter.

I find it essential to limit the thickness of the semiconductor layer (the spacing of the electrodes) to the order of one micron (10,000 Angstroms). 1.5 at such spacing, the field strength is of the order of 10,000 V/cm, several orders of magnitude too low to account for field emission (into vacuum). A falling characteristic beyond 30 kV/cm has been observed† on a cadmium sulfide single crystal 0.4- to 1-mm thick, that effect behaves differently externally. The Cs single crystals I tested between mercury electrodes. However, Cds powder of one micron size, the finest fraction selected by sedimentation in water, shows a typical assistor characteristic. Similarly zinc oxide as single crystal 0.5-mm thick will not conduct at comparable field strength, nor when powdered as to 25-μm grain size: but it behaves like 1-μm powder when further crushed between the electrodes. Observations including tests with mercury electrodes show that during operation no pressure is needed. But initial pressure was needed, mild if merely to seat the 1-μm grains for electrical connection, considerable if larger grains must first be crushed. The electrodes need not be thick; a deposit of 0.1 μm gold on Mylar plastic film does well as one, or both, of the electrodes.

I argue—tentatively—as follows:—1) Unlike metals, semiconductors do not shield their charge against external electric fields. Thus their conductivity can be modified in a region near their surface by the proximity of a conductor outside. As Joffe‡ had predicted and observed, conductivity may be much reduced or increased, depending on electron or hole conduction and on internal electrical fields. Accordingly the conductivity of zinc oxide with the metals I use is increased in a surface region of a thickness estimated to 1 μ. This agrees with the size of grains observed microscopically and with the electrode spacing—1 μ from short to open circuit—measured in a device designed for the test. 2) Only a fraction of a volt may be needed to free a charge in a semiconductor (corresponding to ionization in a gas). 3) If a potential drop of some three times the ionization potential suffices for an avalanche in a gas discharge, one or a few volts may produce a similar breakdown characteristic in a conductor-proximity modified semiconductor. Instructive though they were, powdered semiconductors are now superseded by thin films. Particularly convenient are pairs of oxidized metal electrodes (for symmetry best both the same), such as oxidized iron wire. I heat plain iron wire of 0.010-in diameter in air by passing through it about 2 amperes, to about 450°C as estimated from the increase in resistance, for about 15 to 30 min. Bright gloss changes to dull light grey, a discolored metallic surface as seen under an electron microscope. This film is hard and hard to remove; longer heating yields a black skin that freely peels on touch. Pieces of such wire lightly touching each other appear to be useful at almost every spot and make a remarkably stable resistor. The characteristic has voltage peaks at 2.5-10 (mostly 4 to 6) v at about 0.1 ma, usually very stable, reversibly, affected by heating that lowers the peaks and flattens the dropping resistors.

* Received October 11, 1962.