Portland State UNIVERSITY

Fariborz Maseeh Department of Mathematics and Statistics

MTH 254: Calculus IV

Updated Fall 2021

Description: An introduction to differential and integral calculus of functions of several variables, including vector geometry, the calculus of vector-valued functions, and applications.

Credits: 4

Prerequisite: Mth 253 or (Mth 252 and Mth 261).

Course Objectives: This is the fourth course in the calculus sequence. The course focuses on basic concepts of functions of several variables and vector-valued functions. The theory of differential and integral calculus of functions of one variable is generalized for functions of several variables and vector-valued functions.

Student Learning Outcomes: Upon completion of this course students will:

- Have the ability to visualize vector-valued functions, perform calculus on vector-valued functions, and use vector-valued functions to describe motions of objects in two and three dimensions.
- Understand basic concepts of calculus for functions of several variables such as limits, continuity, partial derivatives, gradient and directional derivatives, and multiple integrals.
- Be able to apply calculus rules in order to compute limits, partial derivatives, gradient and directional derivatives, and multiple integrals for functions of several variables under various operations.
- Have the ability to apply calculus for functions of several variables to solve unconstrained and constrained optimization problems in two and three dimensions.

Topics:

1. *Vector Geometry:* Vectors in two and three dimensions, dot and cross products, planes, quadric surfaces, cylindrical and spherical coordinates.

2. *Calculus of Vector-Valued Functions*: vector-valued functions, calculus of vector-valued Functions, arc length and speed, curvature.

3. *Differentiation in Several Variables:* functions of two or more variables, limits and continuity in several variables, partial derivatives, differentiability, linear approximation, tangent planes, the gradient and directional derivatives, the chain rule, optimization in several variables, Lagrange multipliers.

4. *Multiple Integration:* integration in several variables, double integrals over more general regions, triple integrals, integration in polar, cylindrical, and spherical coordinates, change of variable.

Textbook:

Jon Rogawski, Colin Adams, and Robert Franzosa *Calculus: Early Transcendentals*, 4th ed., Freeman, W.H. & Company 2019.

MTH 254 Textbook Mapping:
Jon Rogawski, Colin Adams, and Robert Franzosa *Calculus: Early Transcendentals*, 4th ed., Freeman, W.H.& Company 2019.

12 Vector Geometry
- 12.1 Vectors in the Plane
- 12.2 Three-Dimensional Space: Surfaces, Vectors, and Curves
- 12.3 Dot Product and the Angle between Two Vectors
- 12.4 The Cross Product
- 12.5 Planes in 3-Space
- 12.6 A Survey of Quadric Surfaces
- 12.7 Cylindrical and Spherical Coordinates

13 Calculus of Vector-Valued Functions
- 13.1 Vector-Valued Functions
- 13.2 Calculus of Vector-Valued Functions
- 13.3 Arc Length and Speed
- 13.4 Curvature

14 Differentiation in Several Variables
- 14.1 Functions of Two or More Variables
- 14.2 Limits and Continuity in Several Variables
- 14.3 Partial Derivatives
- 14.4 Differentiability, Tangent Planes, and Linear Approximation
- 14.5 The Gradient and Directional Derivatives
- 14.6 Multivariable Calculus Chain Rules
- 14.7 Optimization in Several Variables
- 14.8 Lagrange Multipliers: Optimizing with a Constraint

15 Multiple Integration
- 15.1 Integration in Two Variables
- 15.2 Double Integrals over More General Regions
- 15.3 Triple Integrals
- 15.4 Integration in Polar, Cylindrical, and Spherical Coordinates
- 15.6 Change of Variables